Polarity Problems and Solutions..

Polarity is a simple method not really requiring much textual explanation. A brief glance at the section on Polarity Solutions should in most instances be sufficient for understanding. However, a brief discussion may be helpful to those unacquainted with it. Afterwards, go to the Polarity Solutions section on page 60 and look at the examples.

If there are only two candidate squares in any row, column, or box, then one or the other must be the true candidate, so if one is true, the other is false. This oppositeness can be expressed by giving the first candidate a plus sign and the second a minus sign. These two signs signify that one or the other of the candidates is the true candidate, the other a false candidate, the signs being arbitrary, there being no relationship between plus or minus and true or false. If the second candidate square enjoys the same relationship with a third candidate square, in that they are the only two candidate squares in the same row, column, or box, then the third candidate can be assigned the opposite sign to the second candidate. If a fourth candidate shares the same relationship with the third candidate, we have formed a chain of four candidates, each being given the opposite polarity from the preceding candidate (polarity meaning plus or minus). If the first candidate is a plus, and it turns out to be true, then the second candidate, being a minus, is false, the third candidate, being plus, is true, while the fourth candidate, being minus, is false. And if the first plus candidate is false, then the second candidate, being minus is true, the third (plus) candidate is false, and the fourth (minus) candidate is true. By such means we can sometimes construct a chain of such relationships, alternating from plus to minus to plus to minus, and so on. If we ever discover which polarity signifies the candidate to be the correct one, we automatically know the correctness of all the plus and minus candidates in the chain. Either all the plus candidates are true, or all the minus candidates are true.

If a candidate square which is not in the chain happens to share a row, column, or box with a candidate in the chain, we say that it can "see" that candidate. And if a candidate square not in the chain can "see" both a plus candidate in the chain and another, minus candidate in the same chain, then we know that that candidate cannot be a viable one, because it is in the same row, box, or column with both a plus candidate and a minus candidate, one of which is true, and two candidates in the same row, column, or box cannot both be true (i.e. cannot both be viable, since only one candidate can be the correct one in any row, column, or box). Such a candidate, "seeing" both a plus candidate and a minus candidate, must be removed from its square.

This is the principle of polarity chains. It is a way of discovering false candidates. It is easier to understand this argument by looking at examples, so if you are in doubt about the meaning of this principle, it is best to look at the polarity solution section, which explains it better than abstract language can.

Polarity Problem No. 1

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2	5								5
row3	5		5					5	
row4									
row5		5	5	5		5			
row6		5		5		5			
row7				5		5			5
row8	5	5				5		5	
row9		5	5			5			

Polarity Problem No. 2

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					1	1			
row2			1		1			1	
row3			1					1	
row4									
row5									
row6									
row7					1	1			1
row8					1	1			1
row9									

Polarity Problem No. 3

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	**9**		**9**		**9**				**9**
row2	**9**			**9**					**9**
row3	**9**								**9**
row4									
row5									
row6									
row7	**9**			**9**					
row8									
row9	**9**		**9**		**9**				

Polarity Problem No. 4

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		**8**					**8**	**8**	
row2									
row3	**8**							**8**	
row4	**8**							**8**	
row5		**8**					**8**	**8**	
row6									
row7									
row8									
row9									

Polarity Problem No. 5

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4				**4**				**4**	
row5									
row6						**4**			**4**
row7				**4**				**4**	
row8			**4**	**4**				**4**	**4**
row9			**4**			**4**			

Polarity Problem No. 6

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4				**3**	**3**				
row5									
row6									
row7			**3**						**3**
row8				**3**	**3**				
row9			**3**	**3**	**3**				**3**

Polarity Problem No. 7

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		**4**		**4**	**4**	**4**		**4**	
row2		**4**						**4**	
row3				**4**	**4**	**4**			
row4									
row5				**4**		**4**			
row6									
row7				**4**	**4**				
row8									
row9									

Polarity Problem No. 8

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	**2**		**2**		**2**		**2**		
row2	**2**		**2**	**2**					
row3			**2**						**2**
row4									
row5									
row6									
row7	**2**			**2**					**2**
row8				**2**					**2**
row9			**2**		**2**		**2**		**2**

Polarity Problem No. 9

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			**3**				**3**		
row3		**3**	**3**				**3**	**3**	
row4									
row5									
row6									
row7									
row8			**3**				**3**		
row9		**3**	**3**					**3**	

Polarity Problem No. 10

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	**7**		**7**		**7**				
row2	**7**		**7**		**7**				
row3									
row4				**7**		**7**			
row5							**7**	**7**	
row6									
row7			**7**	**7**		**7**	**7**	**7**	
row8	**7**		**7**		**7**		**7**	**7**	
row9			7		7				

Polarity Problem No. 11

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				2	2	2	2		
row2				2	2		2		
row3									
row4	2				2				
row5									
row6	2		2		2	2			
row7									
row8			2		2	2			
row9	2					2			

Polarity Problem No. 12

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4						7		7	
row5	7	7			7	7			
row6		7			7	7			
row7	7	7			7				7
row8						7		7	
row9	7	7			7				7

Polarity Problem No. 13

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1						1	1		
row2				1					1
row3									
row4						1	1		
row5			1	1		1	1		1
row6			1	1			1		
row7									
row8									
row9									

Polarity Problem No. 14

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		8						8	
row2	8						8	8	
row3									
row4									
row5									
row6									
row7									
row8		8						8	
row9	8	8					8	8	

Polarity Problem No. 15

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			8			8	8		
row2	8		8	8		8	8		
row3			8		8		8		8
row4	8			8					
row5			8				8		
row6					8	8			8
row7									
row8									
row9				8		8			

Polarity Problem No. 16

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				1		1	1		
row2						1			1
row3									
row4									
row5									
row6									
row7						1			1
row8									
row9				1		1	1		1

Polarity Problem No. 17

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	3		3		3				
row2	3			3					
row3									
row4	3			3					
row5			3		3				
row6									
row7									
row8									
row9									

Polarity Problem No. 18

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2				4		4		4	4
row3						4			4
row4									
row5						4			4
row6					4		4		
row7									
row8				4	4	4	4	4	
row9				4		4		4	

Polarity Problem No. 19

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		7		7			7		
row2	7	7						7	7
row3		7			7		7		
row4									
row5					7				
row6				7	7				
row7	7						7	7	7
row8	7	7					7	7	7
row9									

Polarity Problem No. 20

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			7	7	7		7		
row2			7	7	7		7	7	
row3	7					7			
row4									
row5	7				7	7			
row6			7		7	7			
row7				7				7	
row8					7	7	7	7	
row9									

Polarity Problem No. 21

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2						**9**			**9**
row3					**9**			**9**	**9**
row4									
row5					**9**			**9**	**9**
row6						**9**		**9**	**9**
row7									
row8									
row9									

Polarity Problem No. 22

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		**1**	**1**	**1**	**1**				
row2									
row3		**1**		**1**					
row4									
row5									
row6									
row7		**1**		**1**					
row8			**1**		**1**				
row9									

Polarity Problem No. 23

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		**1**		**1**	**1**			**1**	
row2			**1**				**1**		
row3				**1**	**1**		**1**		
row4		**1**	**1**						
row5									
row6									
row7									
row8					**1**		**1**		
row9				**1**				**1**	

Polarity Problem No. 24

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4				**1**	**1**		**1**		**1**
row5									
row6				**1**			**1**		**1**
row7									
row8					**1**		**1**		
row9				**1**				**1**	

Polarity Problem No. 25

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2								2
row2									
row3			2				2		
row4	2		2				2		2
row5									
row6	2		2						2
row7									
row8									
row9									

Polarity Problem No. 26

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2	2	2							2
row3		2					2		
row4									
row5									
row6					2	2			
row7		2			2		2		
row8	2	2			2	2	2		2
row9	2				2		2		

Polarity Problem No. 27

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		3	3						
row2									
row3									
row4									
row5		3		3					
row6	3		3		3				
row7	3		3	3					
row8	3		3		3				
row9									

Polarity Problem No. 28

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	4		4		4				
row2						4	4		
row3	4		4				4	4	
row4									
row5					4			4	
row6						4	4		
row7	4		4						
row8									
row9									

Polarity Problem No. 29

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				**3**					**3**
row2			**3**					**3**	
row3	**3**					**3**			
row4	**3**					**3**			
row5			**3**	**3**				**3**	**3**
row6				**3**				**3**	**3**
row7									
row8									
row9									

Polarity Problem No. 30

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			**2**				**2**		
row3		**2**							**2**
row4									
row5		**2**		**2**		**2**			
row6			**2**			**2**			
row7				**2**		**2**	**2**		**2**
row8									
row9				**2**		**2**	**2**		

Polarity Problem No. 31

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			**9**					**9**	
row3	**9**						**9**		**9**
row4		**9**					**9**	**9**	**9**
row5									
row6		**9**						**9**	
row7									
row8	**9**							**9**	**9**
row9			**9**				**9**		

Polarity Problem No. 32

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	**4**		**4**				**4**		
row2			**4**						**4**
row3									
row4		**4**					**4**		**4**
row5			**4**				**4**		
row6									
row7									
row8	**4**	**4**							
row9									

Polarity Problem No. 33

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		7		7		7			
row2	7			7					
row3									
row4					7			7	7
row5									
row6					7				7
row7	7	7		7		7		7	7
row8	7			7		7			7
row9				7		7		7	

Polarity Problem No. 34

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4				3				3	
row5						3		3	3
row6									
row7						3		3	3
row8				3				3	
row9									

Polarity Problem No. 35

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			1				1		
row2		1	1		1				
row3				1	1	1	1		1
row4	1		1		1				
row5	1		1			1			
row6									
row7					1				1
row8	1	1		1			1		
row9		1					1		1

Polarity Problem No. 36

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4			2		2				
row5			2			2	2		
row6			2			2			2
row7									
row8					2		2		
row9						2			2

Polarity Problem No. 37

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4									
row5			**4**				**4**	**4**	
row6		**4**					**4**		
row7			**4**				**4**		
row8									
row9		**4**						**4**	

Polarity Problem No. 38

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					**3**		**3**	**3**	**3**
row2									
row3				**3**			**3**	**3**	
row4			**3**		**3**		**3**	**3**	
row5				**3**					**3**
row6			**3**		**3**		**3**	**3**	
row7									
row8							**3**	**3**	**3**
row9									

Polarity Problem No. 39

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					**3**				**3**
row2	**3**				**3**		**3**		
row3		**3**					**3**		
row4									
row5	**3**	**3**							**3**
row6	**3**						**3**		
row7									
row8									
row9									

Polarity Problem No. 40

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1						**1**	**1**		
row2				**1**					**1**
row3									
row4	**1**	**1**							
row5					**1**				**1**
row6				**1**				**1**	
row7	**1**	**1**							
row8						**1**	**1**		
row9				**1**	**1**			**1**	**1**

Polarity Problem No. 41

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4			7	7					
row5	7			7	7				
row6									
row7			7				7		
row8					7		7		
row9	7			7	7				

Polarity Problem No. 42

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					5		5		5
row2	5								5
row3			5			5			
row4		5							5
row5		5					5		5
row6									
row7			5		5	5			
row8	5				5	5			
row9									

Polarity Problem No. 43

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				**1**	**1**				**1**
row2					**1**		**1**		
row3									
row4									
row5				**1**	**1**		**1**		
row6					**1**		**1**		**1**
row7									
row8									
row9									

Polarity Problem No. 44

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			**1**			**1**			
row2	**1**		**1**	**1**					
row3									
row4	**1**		**1**	**1**					
row5									
row6	**1**		**1**			**1**			
row7									
row8									
row9									

Polarity Problem No. 45

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		8	8				8		
row2		8					8		
row3									
row4									
row5		8			8				
row6	8			8	8				
row7	8				8				
row8			8	8					
row9									

Polarity Problem No. 46

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2					4		4		
row3						4		4	
row4									
row5									
row6									
row7									
row8					4			4	
row9					4	4	4		

Polarity Problem No. 47

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	9						9	9	
row2		9					9		
row3									
row4			9			9			
row5									
row6		9				9			
row7	9		9				9	9	
row8			9					9	
row9									

Polarity Problem No. 48

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1						1		1	1
row2									
row3					1				1
row4									
row5					1	1		1	
row6						1	1		
row7	1						1		
row8									
row9	1						1		1

Polarity Problem No. 49

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		1		1	1			1	
row2			1				1		
row3			1	1	1		1		
row4		1	1						
row5									
row6									
row7									
row8					1		1		
row9				1	1			1	

Polarity Problem No. 50

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2							9		9
row3									
row4					9			9	
row5				9			9		
row6									
row7		9		9				9	
row8				9				9	9
row9		9			9		9	9	9

Polarity Problem No. 51

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					**2**		**2**	**2**	
row2			**2**		**2**		**2**	**2**	
row3	**2**				**2**			**2**	
row4									
row5									
row6							**2**		**2**
row7	**2**							**2**	**2**
row8			**2**					**2**	
row9									

Polarity Problem No. 52

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4									
row5					**1**				**1**
row6					**1**	**1**		**1**	**1**
row7					**1**			**1**	
row8									
row9						**1**			**1**

Polarity Problem No. 53

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				1			1	1	
row2					1			1	1
row3									
row4	1						1		1
row5	1								1
row6									
row7					1				1
row8									
row9				1				1	

Polarity Problem No. 54

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4		5	5		5		5		
row5			5	5	5		5		
row6			5				5		
row7		5		5	5				
row8			5		5				
row9									

Polarity Problem No. 55

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	1	1							1
row2									
row3		1						1	
row4									
row5									
row6									
row7		1		1				1	
row8	1							1	
row9	1	1		1				1	1

Polarity Problem No. 56

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4									
row5		9						9	
row6			9				9		
row7			9		9			9	
row8		9					9		
row9					9		9		

Polarity Problem No. 57

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2	**5**	**5**							**5**
row3		**5**						**5**	
row4							**5**		**5**
row5									
row6									
row7									
row8		**5**					**5**		
row9	**5**							**5**	

Polarity Problem No. 58

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	**9**		**9**		**9**			**9**	
row2					**9**			**9**	
row3	**9**		**9**	**9**					
row4									
row5									
row6									
row7	**9**			**9**					
row8									
row9			**9**		**9**				

Polarity Problem No. 59

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	6							6	
row2		6					6		
row3									
row4	6	6			6			6	
row5	6				6		6		
row6					6		6	6	
row7									
row8									
row9									

Polarity Problem No. 60

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			1	1		1			
row2									
row3		1				1			
row4		1				1			
row5			1	1		1			
row6									
row7									
row8									
row9									

Polarity Problem No. 61

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	5						5	
row2	5	5					5	5	
row3	5		5						
row4	5								5
row5			5				5		
row6									
row7	5								5
row8									
row9		5						5	

Polarity Problem No. 62

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					9		9	9	9
row2									
row3					9	9		9	9
row4		9			9				
row5			9		9	9	9		
row6				9				9	9
row7					9				9
row8				9			9		
row9		9	9						

Polarity Problem No. 63

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			7						7
row3	7							7	7
row4									
row5									
row6									
row7	7								7
row8	7		7	7				7	
row9	7		7	7					

Polarity Problem No. 64

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			7				7	7	
row2									
row3	7	7							7
row4			7						7
row5	7							7	
row6									
row7	7	7						7	
row8									
row9		7	7				7	7	

Polarity Problem No. 65

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4			**6**				**6**		
row5	**6**			**6**		**6**			
row6				**6**			**6**		
row7			**6**			**6**			
row8	**6**							**6**	
row9	**6**			**6**		**6**		**6**	

Polarity Problem No. 66

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			2			2			
row3	2	2	2		2				
row4	2	2			2				
row5			2	2					
row6									
row7									
row8		2	2						
row9				2		2			

Polarity Problem No. 67

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					**4**				**4**
row2				**4**			**4**		
row3									
row4			**4**		**4**			**4**	
row5			**4**	**4**		**4**			
row6					**4**		**4**	**4**	
row7				**4**	**4**	**4**		**4**	
row8									
row9				**4**					**4**

Polarity Problem No. 68

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			4	4		4	4		
row2			4				4		4
row3					4				4
row4						4	4		4
row5					4		4		
row6									
row7				4		4			
row8									
row9									

Polarity Problem No. 69

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			**1**				**1**		
row2									
row3		**1**							**1**
row4									
row5		**1**				**1**			
row6			**1**		**1**	**1**			
row7						**1**			**1**
row8									
row9					**1**	**1**	**1**		**1**

Polarity Problem No. 70

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					2		2		2
row2					2	2			2
row3									
row4						2	2		
row5	2				2				
row6		2							2
row7									
row8	2	2							
row9									

Polarity Problem No. 71

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1						7			7
row2	7	7			7		7	7	
row3	7						7		
row4									
row5									
row6									
row7		7			7		7	7	7
row8		7			7		7	7	7
row9	7					7			

Polarity Problem No. 72

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2	**6**								**6**
row3			**6**					**6**	
row4						**6**	**6**		**6**
row5						**6**	**6**		
row6									
row7			**6**				**6**		
row8				**6**			**6**	**6**	
row9	**6**			**6**				**6**	

Polarity Solutions:

In the solutions which follow, the term "see" is used in the following sense: A square can "see" another square if that other square is in the same row, column, or block. A square can "see" two other squares, for example, if one of the other two squares is in the same row as it, while the other is in the same box, but not in the same common row, or if one is in the same row and the other in the same column. The two other squares need not be able to "see" one another in order that the first square "sees" both of them, although they might. The important fact is that a candidate may not be in the same row, column, or box as a square with an established value equal to it (e.g. a 3-candidate may not be in the same neighborhood as an established 3). It therefore cannot be in the same neighborhood as two other candidates with the same value if it is known that exactly one or the other of those two can be promoted to an established value. If it is, then it must be eliminated as a candidate to its square.

The description of the polarity chains, using double arrows connecting the members in the chain diagram above the solution, is an abstract one, showing only the interconnectedness of its members. It is not to be taken as a geometric description of the relative positions of those squares in the sudoku diagram itself. It is an assist to the shaded squares, which show the positions of the members of the chain, since diagonal arrows are not available to the word processor used to create the diagrams. Its purpose is to show which squares are connected to each other through polarity.

The notation used in this section for the member of a polar pair, where it has a plus or minus attribute, is to express the square in the following way:
"(69+)" refers to square (69), and indicates that it has a plus attribute in the polarity chain. "(15-)" refers to square (15), and indicates that it has a minus attribute in the polarity chain. Since the candidate to be eliminated is not a member of the chain, it has no polarity, so it has no plus or minus attached to it. Thus a candidate to be eliminated, if it were in row 2, column 5, would simply be designated as "(25)".

Polarity Solution No. 1.

Chain:
(21+) ↔ (29-) ↔ (79+)
↕ ↕
(38+) ↔ (88-) (shown in the shaded squares below)

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2	5+								5-
row3	5		5					5+	
row4									
row5		5	5	5		5			
row6		5		5		5			
row7				5		5			5+
row8	~~5~~	5				5		5-	
row9		5	5			5			

(81) can "see" both (21+) & (88-), since it is in the same row as (88) and in the same column as (21), which have opposite polarities, so its 5-candidate must be eliminated, allowing (31-) to be added to the chain, leading to the diagram on the next page.

Note again that the "geometry" of the chain in the arrow diagram at the top of the page is symbolic only. It is used solely for the purpose of showing the polar relationship between squares. It would be nice if it could, but it generally has nothing to do with the actual geometry in the sudoku. It is difficult enough just showing it in a straightforward way, due to the limitations of the word processor used.

Polarity Solution No. 1, continued.

Chain:
(21+) ↔ (29-) ↔ (79+)
↕ ↕ ↕
(31-) (38+) ↔ (88-) (shown in the shaded squares below)

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2	5+								5-
row3	5-		~~5~~					5+	
row4									
row5		5	5	5		5			
row6		5		5		5			
row7				5		5			5+
row8		5				5		5-	
row9		5	5			5			

(33) can "see" both (31-) & (38+), so its 5-candidate must be eliminated as well.

The use of "see" is explained at the beginning of this section.

Polarity Solution No. 2

Chain:
(23+)   (28-)
↕   ↕
(33-) ↔ (38+)   (shown in shaded squares).

Note that squares (23) & (28) are not directly linked, but are connected through (33) & (38). This aspect of a chain "circling" a candidate-to-be-removed is a common one.

(25) can "see" both (23+) & (28-), so its 1-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					1	1			
row2			1+		~~1~~			1-	
row3			1-					1+	
row4									
row5									
row6									
row7					1	1			1
row8					1	1			1
row9									

Polarity Solution No. 3

Chain:
(15+) ↔ (95-)
↕ ↕
(24-) ↔ (74) ↔ (71-) (shown in shaded squares).

Square (11) can "see" both (15+) & (71-), hence its candidate must be eliminated. Note again the way in which the chain "circles around" a candidate-to-be-removed, "attacking" it from two directions at the same time.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	~~9~~		9		9+				9
row2	9			9-					9
row3	9								9
row4									
row5									
row6									
row7	9-			9+					
row8									
row9	9		9		9-				

Polarity Solution No. 4.

Chain:
(12+) ↔ (52-)
↕ ↕
(31-) ↔ (41+) ↔ (48-) (shown in shaded squares).

(18) can "see" (12+) & (48-), so its 8-candidate must be removed.
This allows the chain to be extended (see diagram on next page).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		8+					8	~~8~~	
row2									
row3	8-							8	
row4	8+							8-	
row5		8-					8	8	
row6									
row7									
row8									
row9									

Polarity Solution No. 4, continued.

Chain: ↔↔(12+) ↔ (17-)
↕ ↕ ↕
↕ (31-) ↔ (38+)
↕ ↕ ↕
(52-) ↔(41+) ↔ (48-) (shown in shaded squares).

(58) can "see" both (48-) & (38+), so its 8-candidate must be removed. Equally, it can see both (52-) & (38+).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		8+					8-		
row2									
row3	8-							8+	
row4	8+							8-	
row5		8-					8	~~8~~	
row6									
row7									
row8									
row9									

Polarity Solution No. 5.

Chain:

(44+) ↔ (48-)
↕ ↕
(83+) (66-) ↔ (69+)
↕ ↕ ↕
(93-) ↔ (96+) (89-) (shown in shaded squares).

(84) can "see" both (83+) & (89-). The same is true of (88).
Therefore the 4-candidate must be eliminated from both (84) & (88).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4				4+				4-	
row5									
row6						4-			4+
row7				4				4	
row8			4+	~~4~~				~~4~~	4-
row9			4-			4+			

With these two eliminations, the remaining squares, (74) & (78), may be incorporated into the polarity chain, and then, replacing plus signs by subscript 1's & minus signs by subscript 2's, the pattern analysis is accomplished as well.

Polarity Solution No. 6.

Chain:
(73+) ↔ (79-)
↕ ↕
(93-) (99+) (shown in shaded squares)

Note that (93-) & (99+) are not directly linked.

(94) & (95) can both see (93-) & (93+),
so the 3-candidates of both must be eliminated. Note again the "end-around" "pincer movement" way in which the chain works.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4				3	3				
row5									
row6									
row7			3+						3-
row8				3	3				
row9			3-	~~3~~	~~3~~				3+

Note that the same elimination is obtained by the use of either of the two xwings or the box-line principle in box H.

Polarity Solution No. 7.

Chain:
(12+)   (18-)
  ↕   ↕
(22-) ↔ (28+) (shown in shaded squares)

(14), (15) & (16) can all "see" both (14+) & (18-),
so the 4-candidates in (14), (15) & (16) must be eliminated.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		4+		~~4~~	~~4~~	~~4~~		4-	
row2		4-						4+	
row3				4	4	4			
row4									
row5				4		4			
row6									
row7				4	4				
row8									
row9									

Note that the same elimination can be made by the use of the 2-wing in squares (12), (18), (22), & (28), or by the box-line principle in box B.

Polarity Solution No. 8.

Chain: (33+) ↔ (39-) ↔ (17+) ↔ (97-) (shown in shaded squares)

(93) can "see" both (33+) & (97-). Therefore its 2-candidate must be eliminated.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	~~2~~		2		2		2+		
row2	~~2~~		2	2					
row3			2+						2-
row4									
row5									
row6									
row7	2			~~2~~					~~2~~
row8				2					2
row9			~~2~~		2		2-		2

Note that this causes the 2-candidate in (71) to become unique in box G, so it therefore becomes promoted to the established value of 2, and eliminates the 2-candidates in (21), (11), (74) & (79).

Polarity Solution No. 9.

Chain: (38+) ↔ (98-) ↔ (87+) ↔ (83-) (shown in shaded squares)

(33) can "see" both (38+) & (83-), so its 3-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			3				3		
row3		3	~~3~~				3	3+	
row4									
row5									
row6									
row7									
row8			3-				3+		
row9		3	3					3-	

Polarity Solution No. 10.

Chain: (44+) ↔ (46-)
↕ ↕
(74-) (76+) (shown in shaded squares)

Note that (74-) & (76+) are not directly linked.

All three squares (73), (77) & (78) can "see" both (74-) & (76+),
so the 3-candidate must be removed from all three of them.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	3		3		3				
row2	3		3		3				
row3									
row4				3+		3-			
row5							3	3	
row6									
row7			~~3~~	3-		3+	~~3~~	~~3~~	
row8	3		3		3		3	3	
row9			3		3				

With these 3 squares removed from the 3-pattern, another chain may be established to cause further removals. See the next page.

Polarity Solution No. 10, continued.

Chain: (57+) ↔ (58-)
↕ ↕
(87-) (88+) (shown in shaded squares)

Note that (87-) & (88+) are not directly linked.

All three squares (81), (83) & (85) can "see" both (87-) & (88+).
Therefore the 3-candidate must be removed from all three of them.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	3		3		3				
row2	3		3		3				
row3									
row4				3		3			
row5							3+	3-	
row6									
row7				3		3			
row8	~~3~~		~~3~~		~~3~~		3-	3+	
row9			3		3				

Polarity Solution No. 11. ↔ ↕

Chain: (63+) ↔ (83-) ↔ (91+) ↔ (96-) (shown in shaded squares)

(66) can "see" both (63+) & (96-), so its 3-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				3	3	3	3		
row2				3	3		3		
row3									
row4	3				3				
row5									
row6	3		3+		3	~~3~~			
row7									
row8			3-		3	3			
row9	3+					3-			

Polarity Solution No. 12. ↔ ↕

Chain: (46+) ↔ (48-)
↕
(86-) ↔ (88+) (shown in shaded squares)

Note that (46+) & (86-) not directly linked.

(56) & (66) can both "see" both (46+) & (86-),
so the 3-candidates must be removed from both of them.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4						7+		7-	
row5	7	7			7	~~7~~			
row6		7			7	~~7~~			
row7	7	7			7				7
row8						7-		7+	
row9	7	7			7				7

Polarity Solution No. 13.

Chain: (24+) ↔ (16-)
↕ ↕
(29-) ↔ (17+)
↕
(59+) (shown in shaded squares)

(56) can both "see" both (16-) & (59+), so its 1-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1						1-	1+		
row2				1+					1-
row3									
row4						1	1		
row5			1	1		~~1~~	1		1+
row6			1	1			1		
row7									
row8									
row9									

Polarity Solution No. 14.

Chain: (12+) ↔ (21-) ↔ (91+)
↕
(18-) (shown in shaded squares)

(98) can "see" both (91+) & (18-), so its 8-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		8+						8-	
row2	8-						8	8	
row3									
row4									
row5									
row6									
row7									
row8		8						8	
row9	8+	8					8	~~8~~	

Polarity Solution No. 15.

Chain: (21+) ↔ (41-) ↔ (44+)
↕
(53+) ↔ (57-) ↔ (69+) ↔ (39-) (shown in shaded squares)

Both (27) & (33) can "see" both (39-) & (21+),
so the 8-candidates for both (27) & (33) must be removed.
Note that (27) can also "see" both (57-) & (21+).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			8			8	8		
row2	8+		8	8		8	~~8~~		
row3			~~8~~		8		8		8-
row4	8-			8+					
row5			8+				8-		
row6					8	8			8+
row7									
row8									
row9				8		8			

Polarity Solution No. 16.

Chain: (17+) ↔ (29-) ↔ (26+)
↕
(53-) (shown in shaded squares)

(96) can "see" both (26+) & (97-),
so the 1-candidate for (96) must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				1		1	1+		
row2						1+			1-
row3									
row4									
row5									
row6									
row7						1			1
row8									
row9				1		~~1~~	1-		1

Polarity Solution No. 17.

Chain: (15+) ↔ (55-) ↔ (53+) ↔ (13-)
↕ ↕ ↕
(21+) ↔ (24-) ↔ (44+) ↔ (41-) (shown in shaded squares)

Note that neither (41-) nor (13-) is directly linked to (21+). The only square linked to (21+) is (24-).

(11) can "see" both (15+) & (13-),
so the 8-candidate for (96) must be removed.
Note that (11) can also "see" both (15+) & (41-),
(11) can also "see" both (21+) & (41-), and
(11) can also "see" both (21+) & (13-).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	~~3~~		3-		3+				
row2	3+			3-					
row3									
row4	3-			3+					
row5			3+		3-				
row6									
row7									
row8									
row9									

Once the 3-candidate in square (11) is removed, we can the convert all the plusses to the subscript 1, and all the minuses to the subscript 2, and we have a complete subpattern analysis.

Polarity Solution No. 18.

Chain: (56+) ↔ (59-)
↕ ↕
(65-) ↔ (67+)
↕ ↕
(85+) (87-) (shown in shaded squares)

Note that (85+) & (87-) are not directly linked.

Note that (84), (86) & (88) can all "see" both (85+) & (87-),
so all three must have their 4-candidates removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2				4		4		4	4
row3						4			4
row4									
row5						4+			4-
row6					4-		4+		
row7									
row8				~~4~~	4+	~~4~~	4-	~~4~~	
row9				4		4		4	

After the removal of the 4-candidates from these three squares, the chain may be expanded to allow the removal of one more 4-candidate. See the next page

Polarity Solution No. 18, continued. ↔ ↕

Chain: (56+) ↔ (59-)
↕ ↕
(65-) ↔ (67+)
↕ ↕
(85+) (87-) ↔ (98+) ↔ (28-) (shown in shaded squares)

Now (26) can "see" both (56+) & (28-), so its 4-candidate must be removed as well.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2				4		~~4~~		4-	4
row3						4			4
row4									
row5						4+			4-
row6					4-		4+		
row7									
row8					4+		4-		
row9				4		4		4+	

Polarity Solution No. 19.

Chain: (14+) ↔ (35-)
↕
(64-) ↔ (65+) (shown in shaded squares)

Note that (55) can "see" both (65+) & (35-), so its 7-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		7		7+			7		
row2	7	7						7	7
row3		7			7-		7		
row4									
row5					~~7~~				
row6				7-	7+				
row7	7						7	7	7
row8	7	7					7	7	7
row9									

Polarity Solution No. 20.

Chain: (63+) ↔ (51-) ↔ (31+) ↔ (36-) (shown in shaded squares)

Note that (66) can "see" both (63+) & (36-), so its 7-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			7	7	7		7		
row2			7	7	7		7	7	
row3	7+					7-			
row4									
row5	7-				7	7			
row6			7+		7	~~7~~			
row7				7				7	
row8					7	7	7	7	
row9									

Polarity Solution No. 21

Chain: (35+) ↔ (26-) ↔ (29+)
↕ ↕
(55-) ↔ (66+) (shown in shaded squares)

Note that (59) can "see" both (29+) & (55-),
so its 9-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2						9-			9+
row3					9+			9	9
row4									
row5					9-			9	~~9~~
row6						9+		9	9
row7									
row8									
row9									

Polarity Solution No. 22

Chain: (13+) ↔ (83-) ↔ (72+)
↕ ↕
(15-) ↔ (85+) ↔ (74-) (shown in shaded squares)

Note that both (12) & (14) can "see" both (13+) & (15-),
so the 1-candidates from both (12) & (14) must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		~~1~~	1+	~~1~~	1-				
row2									
row3		1		1					
row4									
row5									
row6									
row7		1+		1-					
row8			1-		1+				
row9									

Polarity Problem No. 23

Chain: (18+) ↔ (98-) ↔ (87+)
↕ ↕
(94+) ↔ (85-) (shown in shaded squares)

(15) can "see" both (18+) & (85-), so its 1-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		**1**		**1**	~~**1**~~			**1+**	
row2			**1**				**1**		
row3				**1**	**1**		**1**		
row4		**1**	**1**						
row5									
row6									
row7									
row8					**1-**		**1+**		
row9				**1+**				**1-**	

Polarity Solution No. 24

Chain: (45+) ↔ (85-) ↔ (87+)
↕ ↕
(94+)↔ (99-) (shown in shaded squares)

(49) can "see" both (45+) and (99-), so its 1-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4				**1**	**1+**		**1**		~~**1**~~
row5									
row6				**1**			**1**		**1**
row7									
row8					**1-**		**1+**		
row9				**1+**					**1-**

Polarity Solution No. 25

Chain: (11+) ↔ (19-)
↕ ↕
(33-) ↔ (37+) ↔ (47-) (shown in shaded squares)

(41) can "see" both (11+) & (47-), so its 2-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	**2+**								**2-**
row2									
row3			**2-**				**2+**		
row4	~~**2**~~		**2**				**2-**		**2**
row5									
row6	**2**		**2**						**2**
row7									
row8									
row9									

Polarity Solution No. 26

Chain: (29+) ↔ (89-)
↕
(37-) ↔ (32+) (shown in shaded squares)

(82) can "see" both (89-) & (32+), so its 2-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2	2	2							2+
row3		2+					2-		
row4									
row5									
row6					2	2			
row7		2			2		2		
row8	2	~~2~~			2	2	2		2-
row9	2				2		2		

Polarity Solution No. 27

Chain: (12+) ↔ (52-) ↔ (54+) ↔ (65-)
↕ ↕ ↕
(13-) (74-) ↔ (85+) (shown in shaded squares)

(83) can "see" both (13-) & (85+), so its 3-candidate must be erased.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		**3+**	**3-**						
row2									
row3									
row4									
row5		**3-**		**3+**					
row6	**3**		**3**		**3-**				
row7	**3**		**3**	**3-**					
row8	**3**		~~**3**~~		**3+**				
row9									

Polarity Solution No. 28

Chain: (15+) ↔ (26-) ↔ (27+)
↕ ↕
(55-) ↔ (58+) ↔ (38-)
↕ ↕
(66+) ↔ (67-) (shown in shaded squares)

(37) can "see" both (27+) & (38-), so its 4-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	4		4		4+				
row2						4-	4+		
row3	4		4				~~4~~	4-	
row4									
row5					4-			4+	
row6						4+	4-		
row7	4		4						
row8									
row9									

Polarity Solution No. 29

Chain: (14+) ↔ (19-) ↔ (28+)
↕ ↕
(36-) ↔ (31+) ↔ (23-)
↕ ↕ ↕
(46+) ↔ (41-) ↔ (53+) (shown in shaded squares)

(59) can "see" both (53+) & (19-), so its 3-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				**3+**					**3-**
row2			**3-**					**3+**	
row3	**3+**					**3-**			
row4	**3-**					**3+**			
row5			**3+**	**3**				**3**	~~**3**~~
row6				**3**				**3**	**3**
row7									
row8									
row9									

Polarity Solution No. 30

Chain: (27-) ↔ (39+) ↔ (79-)
↕ ↕ ↕
(23+) ↔ (32-)
↕ ↕
(66+) ↔ (63-) ↔ (52+) (shown in shaded squares)

(76) can "see" both (66+) & (79-), so its 2-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			**2+**				**2-**		
row3		**2-**							**2+**
row4									
row5		**2+**		**2**		**2**			
row6			**2-**			**2+**			
row7				**2**		~~**2**~~	**2**		**2-**
row8									
row9				**2**		**2**	**2**		

Polarity Solution No. 31

Chain: (31-) ↔ (23+) ↔ (28-)
↕ ↕ ↕
(81+) ↔ (93-) ↔ (97+) (shown in shaded squares)

(37) can "see" both (28-) & (97+), so its 9-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			**9+**					**9-**	
row3	**9-**						~~**9**~~		**9**
row4		**9**					**9**	**9**	**9**
row5									
row6		**9**						**9**	
row7									
row8	**9+**							**9**	**9**
row9			**9-**				**9+**		

After the removal of the 9-candidate from (37), further links in the polarity chain are created, allowing the removal of the 9-candidate from another square. See next page.

Polarity Solution No. 31, continued.

Chain: ↔↔ (39+) ↔↔

↕ ↕ ↕

(31-) ↔ (23+) ↔ (28-) (shown in shaded squares)

↕ ↕

(81+) ↔ (93-) ↔ (97+) ↔ (47-)

(49) can "see" both (39+) & (47-), so its 9-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			9+					9-	
row3	9-								9+
row4		9					9-	9	~~9~~
row5									
row6		9						9	
row7									
row8	9+							9	9
row9			9-				9+		

After the removal of the 9-candidate from (49), further links in the polarity chain are created, allowing the removal of the 9-candidate from yet another square. See next page.

Polarity Solution No. 31, continued.

Chain: (89-)
↕
↔↔ (39+) ↔↔
↕ ↕ ↕
(31-) ↔ (23+) ↔ (28-) (shown in shaded squares)
↕ ↕
(81+) ↔ (93-) ↔ (97+) ↔ (47-)

(88) can "see" both (81+) & (89-), so its 9-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			9+					9-	
row3	9-								9+
row4		9					9-	9	
row5									
row6		9						9	
row7									
row8	9+							~~9~~	9-
row9			9-				9+		

Polarity Solution No. 32

Chain: (11+) ↔ (81-) ↔ (82+) ↔ (42-) ↔ (53+) ↔ (57-)
(shown in shaded squares)

(17) can "see" both (11+) & (57-), so its 4-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	4+		4				~~4~~		
row2			4						4
row3									
row4		4-					4		4
row5			4+				4-		
row6									
row7									
row8	4-	4+							
row9									

Polarity Solution No. 33

Chain: (12+) ↔ (21-)
↕ ↕
(72-) (24+) (shown in shaded squares)

(74) can "see" both (72-) & (24+), so its 7-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		7+		7		7			
row2	7-			7+					
row3									
row4					7			7	7
row5									
row6					7				7
row7	7	7-		~~7~~		7		7	7
row8	7			7		7			7
row9				7		7		7	

Polarity Solution No. 34

Chain: (56-) ↔ (44+) ↔ (48-)
↕ ↕
(76+) ↔ (84-) ↔ (88+) (shown in shaded squares)

Both (58) & (78) can "see" both (48-) & (88+), so the 3-candidates of both (58) & (78) must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4				**3+**				**3-**	
row5						**3-**		~~**3**~~	**3**
row6									
row7						**3+**		~~**3**~~	**3**
row8				**3-**				**3+**	
row9									

Polarity Solution No. 35

Chain: (34+) ↔ (84-) ↔ (75+) ↔ (79-)(shown in shaded squares)

(39) can "see" both (34+) & (79-), so its 1-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			1				1		
row2		1	1		1				
row3				1+	1	1	1		~~1~~
row4	1		1		1				
row5	1		1			1			
row6									
row7					1+				1-
row8	1	1		1-			1		
row9		1					1		1

Polarity Solution No. 35, continued.

After the removal of the 1-candidate from (39), the chain must be extended to: (34+) ↔ (84-) ↔ (75+) ↔ (79-) ↔ (99+)   (shown in shaded squares)

Both (87) & (97) can "see" both (99+) & (79-), so the 1-candidates from both (87) & (97) must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			1				1		
row2		1	1		1				
row3				1+	1	1	1		
row4	1		1		1				
row5	1		1			1			
row6									
row7					1+				1-
row8	1	1		1-			~~1~~		
row9		1					~~1~~		1+

Polarity Solution No. 36

Chain: (43+) ↔ (45-) ↔ (85+) ↔ (87-) ↔ (57+)
↕ ↕ ↕
(96-) ↔ (99+) ↔ (69-) (shown in shaded squares)

(56) can "see" both (45-) & (57+), so its 1-candidate must be removed.
(63) can "see" both (43+) & (69-), so its 1-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4			**2+**		**2-**				
row5			**2**			~~**2**~~	**2+**		
row6			~~**2**~~			**2**			**2-**
row7									
row8					**2+**		**2-**		
row9						**2-**			**2+**

Polarity Solution No. 37

This is an unusual polarity problem, in that there are two 4-pluses in the same row, as well as two 4-pluses in the same column, and two 4-pluses in the same box. Clearly, the 4-pluses are all all false, and must all be eliminated as candidates. See the next page for the final situation.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4									
row5			**4+**				**4**	**4+**	
row6		**4-**					**4+**		
row7			**4-**				**4+**		
row8									
row9		**4+**						**4-**	

Note that the 4 pluses are all killer candidates. This is one of many crossover situations, in some of which box-line solutions are equivalent to n-wing solutions, in others killer candidates must also be eliminated by n-wing procedures, and in others, like this one, an analysis of the subpatterns reveals the 4+ candidates to be killer candidates.

Polarity Solution No. 37, continued.

This is the final outcome of the reduction through polarity. As indicated on the preceding page, the same result could be obtained through subpattern analysis.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4									
row5							4		
row6		4							
row7			4						
row8									
row9								4	

Polarity Solution No. 38

Chain: (15+) ↔ (34-) ↔ (54+) ↔ (59-) (shown in shaded squares)

(19) can "see" both (15+) & (59-), so its 3-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					3+		3	3	~~3~~
row2									
row3				3-			3	3	
row4			3		3		3	3	
row5				3+					3-
row6			3		3		3	3	
row7									
row8							3	3	3
row9									

Polarity Solution No. 39

Chain: (15+) ↔ (19-) ↔ (59+) ↔ (67-)
↕ ↕
(25-) (61+) (shown in shaded squares)

(21) can "see" both (25-) & (61+), so its 3-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					**3+**				**3-**
row2	~~**3**~~				**3-**		**3**		
row3		**3**					**3**		
row4									
row5	**3**	**3**							**3+**
row6	**3+**						**3-**		
row7									
row8									
row9									

Polarity Solution No. 40

Chain: (95-) ↔ (55+) ↔ (59-)
↕ ↕
(64-) ↔ (68+) (shown in shaded squares)

(98) can "see" both (68+) & (95-), so its 1-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1						1	1		
row2				1					1
row3									
row4	1	1							
row5					1+				1-
row6				1-				1+	
row7	1	1							
row8						1	1		
row9				1	1-			~~1~~	1

see the next two pages for the aftermath to this situation.

Polarity Solution No. 40, continued.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1						1	1		
row2				1					1
row3									
row4	1	1							
row5					1				~~1~~
row6				~~1~~				1	
row7	1	1							
row8						1	1		
row9				1	~~1~~				1

Removing the 1-candidate from (98), the 1-candidate in column 8 is now unique, so it becomes an established 1 (shown in large font), which then causes the 1-candidates in (64) & (59) to be eliminated, causing the remaining 1-candidate in (55) to become an established 1 (also shown in large font), which then causes the 1-candidate in 95) to be eliminated. See the next page for the final pattern.

Polarity Solution No. 40, continued.

This is the final 1-pattern. The eliminated 1-candidates & the promoted 1-candidates are, of course, not shown, since they are no longer part of the pattern of 1-candidates:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1						**1**	**1**		
row2				**1**					**1**
row3									
row4	**1**	**1**							
row5									
row6									
row7	**1**	**1**							
row8						**1**	**1**		
row9				**1**					**1**

Polarity Solution No. 41

Chain: (51-) ↔ (43+) ↔ (44-)
↕ ↕
(91+) ↔ (73-) ↔ (77+) ↔ (87-) ↔ (85+) (shown in shaded squares)

(55) can "see" both (51-) & (85+), so its 7-candidate must be removed.
(94) can "see" both (91+) & (44-), so its 7-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4			7+	7-					
row5	7-			7	~~7~~				
row6									
row7			7-				7+		
row8					7+		7-		
row9	7+			~~7~~	7				

Polarity Solution No. 42↔↕

Chain: (15+) ↔ (36-) ↔ (33+) ↔ (21-) ↔ (29+)
↕ ↕
(73-) ↔ (81+) (shown in shaded squares)

(75) can "see" both (15+) & (73-), so its 5-candidate must be removed.
(86) can "see" both (81+) & (36-), so its 5-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					5+		5		5
row2	5-								5+
row3			5+			5-			
row4		5							5
row5		5					5		5
row6									
row7			5-		~~5~~	5			
row8	5+				5	~~5~~			
row9									

Polarity Solution No. 43

Chain: (25+) ↔ (27-) ↔ (19+) ↔ (69-)   (shown in shaded squares)

(65) can "see" both (25+) & (69-), so its 1-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				**1**	**1**				**1+**
row2					**1+**		**1-**		
row3									
row4									
row5				**1**	**1**		**1**		
row6					~~**1**~~		**1**		**1-**
row7									
row8									
row9									

After this removal of the 1-candidate from (65), a further link in the polarity chain is created, allowing the removal of the 1-candidate from another square. See next page.

Polarity Solution No. 43, continued.

Chain: (25+) ↔ (27-) ↔ (19+) ↔ (69-) ↔ (67-)   (shown in shaded squares)

(57) can "see" both (67+) & (27-), so its 1-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				1	1				1+
row2					1+		1-		
row3									
row4									
row5				1	1		~~1~~		
row6							1+		1-
row7									
row8									
row9									

Polarity Solution No. 44

Chain: (13+) ↔ (16-) ↔ (24+)
↕ ↕
(66+) ↔ (44-) (shown in shaded squares)

(43) can "see" both (13+) & (44-), so its 1-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			**1+**			**1-**			
row2	**1**		**1**	**1+**					
row3									
row4	**1**		~~**1**~~	**1-**					
row5									
row6	**1**		**1**			**1+**			
row7									
row8									
row9									

Polarity Solution No. 45

Chain: (13+) ↔ (83-) ↔ (71+)↔ (61-) ↔ (52+)
↕ ↕ ↕
(84+) ↔ (75-) (55-)
↕
(64-) (shown in shaded squares)

This is like polarity problem #37, in that (61-) & (64-), both being in row 6, can both "see" one another, and also (64-) & (55-), being in the same box, can both "see" one another. Clearly the 8's with negative polarity must all be false, and must all be removed. See the next page for the aftermath of their removal.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		**8**	**8+**				**8**		
row2		**8**					**8**		
row3									
row4									
row5		**8+**			**8-**				
row6	**8-**			**8-**	**8**				
row7	**8+**				**8-**				
row8			**8-**	**8+**					
row9									

continued on the next page

Polarity Solution No. 45, continued.

After removal of the 8-'s, we have the situation below. Since the 8's in (52), (65), (71) & (84) are the only 8's remaining in their boxes, they must all be promoted to established 8's. Since the 8 in (13) is the only 8 remaining in column 3, it must also be promoted to an established 8. See the next page for the results of these promotions.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		**8**	**8+**				**8**		
row2		**8**					**8**		
row3									
row4									
row5		**8+**							
row6					**8**				
row7	**8+**								
row8				**8+**					
row9									

continued on the next page

Polarity Solution No. 45, continued.

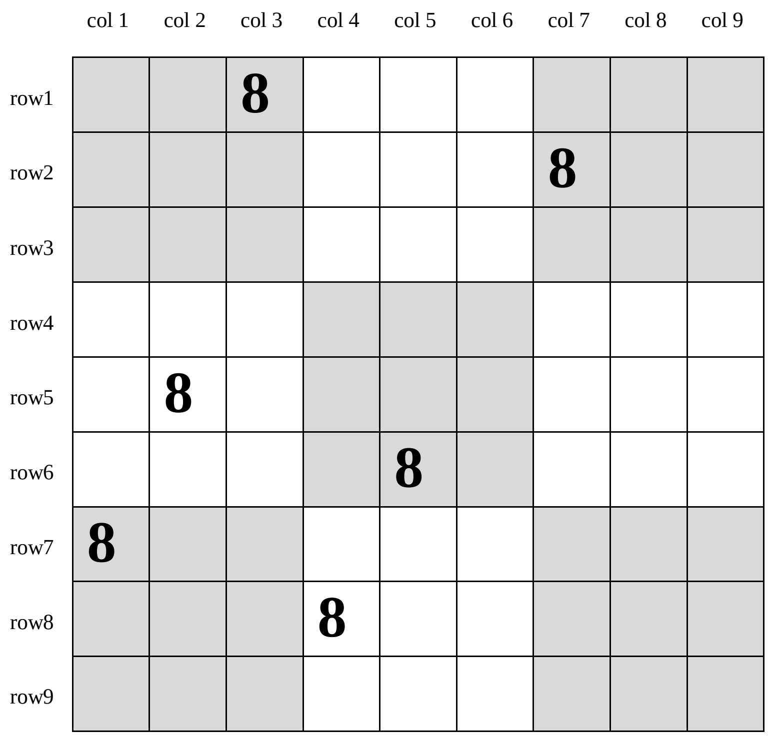
The promotions specified on the preceding page have been carried out, the large 8's below representing established 8's. These established 8's will then eliminate all candidates in their neighborhoods. See the next page for the results of these eliminations.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		8	**8**				8		
row2		8					8		
row3									
row4									
row5		**8**							
row6					**8**				
row7	**8**								
row8				**8**					
row9									

continued on the next page

Polarity Solution No. 45, continued.

After elimination of candidate 8's from the neighborhoods of established 8's, the 8 in (27) is the only candidate left in row 2 (as well as the only candidate in box 13 & column 7), so it must also be promoted to an established 8:



In this situation, we have not only eliminated candidates using polarity. We have eliminated all the candidates which could not become established 8's, and then promoting the remaining candidates to established values. If only polarity were always so powerful a tool!

Note: It may puzzle you that boxes B, F & I do not appear to have established 8's in them. They do, in fact, but they were not shown in the original 8-pattern because patterns consist only of candidates, so they are not necessary, and not shown in this diagram either.

Polarity Solution No. 46

Chain: (25+) ↔ (27-)   (85+)
↕   ↕   ↕
(36-) ↔ (38+) ↔ (88-)
↕   ↕
(96+)   (97+)   (shown in shaded squares)

This is like polarity problems #37 & #45, in that squares (25) & (85) both have positive polarity and are in the same column. Likewise, squares (96) & (97) both have positive polarity and are in the same row. This means that all the 8's with positive polarity are false, so the 8-candidates must therefore be removed from squares (25), (38), (85), (96) & (97). The results of this removal are shown on the next page.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2					**4+**		**4-**		
row3						**4-**		**4+**	
row4									
row5									
row6									
row7									
row8					**4+**			**4-**	
row9					**4**	**4+**	**4+**		

continued on the next page

Polarity Solution No. 46, continued.

The remaining 4-candidates are unique in their neighborhoods, so all of them must be promoted to established values, indicated by the larger font size:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2							**4**		
row3						**4**			
row4									
row5									
row6									
row7									
row8								**4**	
row9					**4**				

Polarity Solution No. 47

Chain: (11+) ↔ (22-) ↔ (27+)
↕ ↕
(71-) (62+) ↔ (66-)
↕ ↕
(43-) ↔ (46+) (shown in shaded squares)

(77) can "see" both (27+) & (71-), so its 9-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	**9+**						**9**	**9**	
row2		**9-**					**9+**		
row3									
row4			**9-**			**9+**			
row5									
row6		**9+**				**9-**			
row7	**9-**		**9**				~~**9**~~	**9**	
row8			**9**					**9**	
row9									

continued on the next page

Polarity Solution No. 47, continued.

With the elimination of the 9-candidate from (77), the chain must be extended:

Chain: (11+) ↔ (22-) ↔ (27+)↔ (17-)
↕ ↕
(71-) (62+) ↔ (66-)
↕ ↕
(43-) ↔ (46+) (shown in shaded squares)

(18) can "see" both (11+) & (17-), so its 9-candidate must be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	**9+**						**9-**	~~**9**~~	
row2		**9-**					**9+**		
row3									
row4			**9-**			**9+**			
row5									
row6		**9+**				**9-**			
row7	**9-**		**9**					**9**	
row8			**9**					**9**	
row9									

Polarity Solution No. 48

Chain: (18+) ↔ (58-) ↔ (67+)↔ (66-)   (shown in shaded squares)

(16) can "see" both (18+) & (66-), so its 1-candidate must be eliminated.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1						~~1~~		**1+**	**1**
row2									
row3					**1**				**1**
row4									
row5					**1**	**1**		**1-**	
row6						**1-**	**1+**		
row7	**1**						**1**		
row8									
row9	**1**						**1**		**1**

continued on the next page

Polarity Solution No. 48, continued.

With the elimination of the 1-candidate in (16), the 1-candidate in (35) becomes unique in box B, so it must be promoted to an established value, indicated by an enlarged font size. This newly established 1 then eliminates all 1-candidates in its neighborhood, in this case the 1-candidates in squares (39) & (55). Although the chain must be further extended after these removals, no further eliminations are possible.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1								1+	1
row2									
row3					1				~~1~~
row4									
row5					~~1~~	1		1-	
row6						1-	1+		
row7	1						1		
row8									
row9	1						1		1

Polarity Solution No. 49

Chain: (18+) ↔ (98-) ↔ (87+)↔ (85-)   (shown in shaded squares)

(15) can "see" both (18+) & (85-), so its 1-candidate must be eliminated.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		1		1	~~1~~			1+	
row2			1				1		
row3			1	1	1		1		
row4		1	1						
row5									
row6									
row7									
row8					1-		1+		
row9				1	1			1-	

Polarity Solution No. 50

Chain: (95-) ↔ (45+) ↔ (48-)
↕ ↕
(54-) ↔ (57+) (shown in shaded squares)

(97) can "see" both (57+) & (95-), so its 9-candidate must be eliminated.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2							9		9
row3									
row4					9+			9-	
row5				9-			9+		
row6									
row7		9		9				9	
row8				9				9	9
row9		9			9-		~~9~~	9	9

continued on the next page

Polarity Solution No. 50, continued.

With the 9-candidate in (97) eliminated, the chain may be extended:

Chain: (95-) ↔ (45+) ↔ (48-)
↕ ↕
(54-) ↔ (57+) ↔ (27-) ↔ (29+)

(shown in shaded squares)

(99) can "see" both (29+) & (95-), so its 9-candidate must be eliminated.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2							9-		9+
row3									
row4					9+			9-	
row5				9-			9+		
row6									
row7		9		9				9	
row8				9				9	9
row9		9			9-			9	~~9~~

Polarity Solution No. 51

Chain: (31+) ↔ (13-)
↕
(71-) ↔ (83+) ↔ (88-) (shown in shaded squares)

(38) can "see" both (31+) & (88-), so its 2-candidate must be eliminated.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					2		2	2	
row2			2-		2		2	2	
row3	2+				2			~~2~~	
row4									
row5									
row6							2		2
row7	2-							2	2
row8			2+					2-	
row9									

Polarity Solution No. 52

Chain: (75+) ↔ (78-) ↔ (68+)
↕ ↕
(71-) ↔ (83+) ↔ (88-) (shown in shaded squares)

This is like polarity problems #37, #45, & #46 in that squares (66) & (68) both have positive polarity and are in the same row. This means that all the 1's with positive polarity are false, so the 1-candidates must therefore be removed from squares (66), (68), (75) & (99). The results of this removal are shown on the next page.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4									
row5					**1**				**1**
row6					**1**	**1+**		**1+**	**1**
row7					**1+**			**1-**	
row8									
row9						**1-**			**1+**

Polarity Solution No. 52, continued.

After removing the 1-candidates from squares (66), (68), (75) & (99), the 1-candidates in squares (78) & (96) are the only 1-candidates in boxes H & I, so these two candidates must both be promoted to established values, indicated by the large digit 1's. Only four 1-candidates remain, in squares (55), (59), (65) & (69).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4									
row5					1				1
row6					1				1
row7								1	
row8									
row9						1			

Polarity Solution No. 53

Chain: (14+) ↔ (94-) ↔ (98+)
↕ ↕ ↕
(25-) ↔ (75+) ↔ (79-) (shown in shaded squares)

Square (28) can "see" both (25-) & (98+), so its 1-candidate may be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				1+			1	1	
row2					1-			~~1~~	1
row3									
row4	1						1		1
row5	1								1
row6									
row7					1+				1-
row8									
row9				1-				1+	

continued on the next page

Polarity Solution No. 53, continued.

With square (28) out of the picture, the chain may be extended:

Chain: (14+) ↔ (94-) ↔ (98+) ↔ (18-)
↕ ↕ ↕
(25-) ↔ (75+) ↔ (79-) (shown in shaded squares)

Square (17) can "see" both (14+) & (18-), so its 1-candidate may be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				**1+**			~~**1**~~	**1-**	
row2					**1-**				**1**
row3									
row4	**1**						**1**		**1**
row5	**1**								**1**
row6									
row7					**1+**				**1-**
row8									
row9				**1-**				**1+**	

continued on the next page.

Polarity Solution No. 53, continued.

With the elimination of the 1-candidate from square (17), the 1-candidate in square (47) becomes unique in column 7, so it must be promoted to an established value (shown in large font). It then eliminates the 1-candidates from all other squares in its neighborhood, namely squares (41), (49) & (59). This results in the 1-candidate in square (51) being unique in box D, so it also must be promoted to an established value.

Chain: (14+) ↔ (94-) ↔ (98+) ↔ (18-)
↕ ↕ ↕
(25-) ↔ (75+) ↔ (79-) (shown in shaded squares)

Square (17) can "see" both (14+) & (18-), so its 1-candidate may be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				1+				1-	
row2					1-				1
row3									
row4	~~1~~						1		~~1~~
row5	1								~~1~~
row6									
row7					1+				1-
row8									
row9				1-				1+	

continued on the next page

Polarity Solution No. 53, continued.

Now the chain can be extended to include square (29), but there is nothing left to eliminate from the 1-pattern.

Chain: (14+) ↔ (94-) ↔ (98+) ↔ (18-)
↕ ↕ ↕ ↕
(25-) ↔ (75+) ↔ (79-) ↔ (29+) (shown in shaded squares)

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				1+				1-	
row2					1-				1+
row3									
row4							1		
row5	1								
row6									
row7					1+				1-
row8									
row9				1-				1+	

Polarity Solution No. 54

Chain: (42+) ↔ (72-) ↔ (83+) ↔ (85-) (shown in shaded squares)

Square (45) can "see" both (42-) & (85-), so its 5-candidate may be removed

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4		5+	5		~~5~~		5		
row5			5	5	5		5		
row6			5				5		
row7		5-		5	5				
row8			5+		5-				
row9									

continued on the next page

Polarity Solution No. 54, continued.

After removing the 5-candidate from square (45), only two 5-candidates remain in box E, both in row 5. They then "own" the row, and the two other 5-candidates in row 5, in squares (53) & (57), which are in the same row but in adjacent boxes may now be removed as well.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4		5+	5				5		
row5			~~5~~	5	5		~~5~~		
row6			5				5		
row7		5-		5	5				
row8			5+		5-				
row9									

Polarity Solution No. 55

Chain: (32+) ↔ (38-) ↔ (19+) ↔ (99-) (shown in shaded squares)

Square (92) can "see" both (19+) & (99-), so its 5-candidate may be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	**1**	**1**							**1+**
row2									
row3		**1+**						**1-**	
row4									
row5									
row6									
row7		**1**		**1**				**1**	
row8	**1**							**1**	
row9	**1**	~~**1**~~		**1**				**1**	**1-**

Polarity Solution No. 56

Chain: (73+) ↔ (82-) ↔ (52+) ↔ (58-) ↔ (78+)
↕ ↕ ↕
(87+) (63-) ↔ (67+) (shown in shaded squares)

Squares (67+) & (87+) can both "see" one another, as can squares (78+) & (87+). Therefore the 9+'s must all be false, and only the 9-'s true. We may therefore eliminate all the 9+'s. See the next page.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4									
row5		**9+**						**9-**	
row6			**9-**				**9+**		
row7			**9+**		**9**			**9+**	
row8		**9-**					**9+**		
row9					**9**		**9**		

Polarity Solution No. 56, continued.

The 9-candidate in square (75) is the only 9-candidate in row 6, so it must be promoted to an established 9, and the 9-candidate in square (97) is unique in box I, so it also must be promoted to an established 9. The 9-candidate in square (95) must therefore be removed. All the other 9-candidates are unique in their rows, columns & boxes, hence must also be promoted to established 9's.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4									
row5								9	
row6			9						
row7					9				
row8		9							
row9					~~9~~		9		

Polarity Solution No. 57

Chain: (32+) ↔ (38-) ←←←←→→→→(98+)↔ (91-)↔ (21+)
↕ ↕ ↕
(29+) ↔(49-) ↔ (47+) ↔(87-)↔ (82+)
(shown in shaded squares)

Squares (32+) & (82+) can both "see" each other. Therefore the 5-candidates in the plus squares must all be false, and only the 5-candidates in the minus squares can be correct. Eliminating the 5-candidates in the plus squares results in the 5-pattern on the next page.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2	**5+**	**5**							**5+**
row3		**5+**						**5-**	
row4							**5+**		**5-**
row5									
row6									
row7									
row8		**5+**					**5-**		
row9	**5-**							**5+**	

continued on the next page

Polarity Solution No. 57, continued.

Since the remaining 5-candidates are all now unique in their rows, columns, and boxes, they must all be promoted to established values, hence they are shown in large font, no longer being candidates.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2		**5**							
row3								**5**	
row4									**5**
row5									
row6									
row7									
row8							**5**		
row9	**5**								

Polarity Solution No. 58

Chain: (71+) ↔ (74-) ↔ (34+)
↕ ↕
(93-) ↔ (95+) (shown in shaded squares)

Square (33) can "see" both (34+) & (93-), so its 9-candidate may be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	9		9		9			9	
row2					9			9	
row3	9		~~9~~	9+					
row4									
row5									
row6									
row7	9+			9-					
row8									
row9			9-		9+				

continued on next page

Polarity Solution No. 58

Chain: (31-) ↔ (34+)
↕ ↕
(71+) ↔ (74-)
↕ ↕
(93-) ↔ (95+) (shown in shaded squares)

Getting rid of the 9-candidate in square (34) allows the addition of square (31-) to the chain, which then permits the removal of the 9-candidate in square (11), which can now "see" both squares (31-) & (71+).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	~~9~~		9		9			9	
row2					9			9	
row3	9-			9+					
row4									
row5									
row6									
row7	9+			9-					
row8									
row9			9-		9+				

Polarity Solution No. 59

Chain: (11+) ↔ (18-)
↕ ↕
(22-) ↔ (27+)
↕
(42 +) (shown in shaded squares)

Square (48) can "see" both (42+) & (18-), so its 6-candidate may be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	6+							6-	
row2		6-					6+		
row3									
row4	6	6+			6			~~6~~	
row5	6				6		6		
row6					6		6	6	
row7									
row8									
row9									

Polarity Solution No. 60

Chain: (13+) ↔ (32-) ↔ (36+)
↕ ↕
(53-) ↔ (42+) ↔ (46-) (shown in shaded squares)

Squares (16) & (56) can "see" both (36+) & (46-), so their 1-candidates may be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			1+	1		~~1~~			
row2									
row3		1-				1+			
row4		1+				1-			
row5			1-	1		~~1~~			
row6									
row7									
row8									
row9									

Polarity Solution No. 61

Chain: (33+) ↔ (53-) ↔ (41+)   (71-) ↔ (92+)
↕ ↕ ↕ ↕
(27-) ↔ (57+) ↔ (49-) ↔ (79+) ↔ (98-) (shown in shaded squares)

Squares (11), (21) & (31) can "see" both (41+) & (71-),
while squares (21) & (22) can "see" both (33+) & (27-),
so their 1-candidates should be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	~~5~~	5						5	
row2	~~5~~	~~5~~					5-	5	
row3	~~5~~		5+						
row4	5+								5-
row5			5-				5+		
row6									
row7	5-								5+
row8									
row9		5+						5-	

continued on the next page

Polarity Solution No. 61, continued.

After removal of the 1-candidates, the diagram is simplified, and the chain is extended:

Chain: (12-) ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔
↕ ↕
(33+) ↔ (53-) ↔ (41+) (71-) ↔ (92+)
↕ ↕ ↕ ↕
(27-) ↔ (57+) ↔ (49-) ↔ (79+) ↔ (98-) (shown in shaded squares)

Square (28) can "see" both (18+) & (27-), so its 1-candidate should be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		5-						5+	
row2							5-	~~5~~	
row3			5+						
row4	5+								5-
row5			5-				5+		
row6									
row7	5-								5+
row8									
row9		5+						5-	

Polarity Solution No. 62.

Chain: (64+) ↔ (84-) ↔ (87+)
↕ ↕
(75+) ↔ (79-) (shown in shaded squares)

Square (69) can "see" both (64+) & (79-), so its 9-candidate should be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					9		9	9	9
row2									
row3					9	9		9	9
row4		9			9				
row5			9		9	9	9		
row6				9+				9-	~~9~~
row7					9+				9-
row8				9-			9+		
row9		9	9						

problem 62 continued on next page

Polarity Solution No. 62, continued.

After removal of the 9-candidate from square (69), the diagram is simplified, and the chain is extended:

Chain: (64+) ↔ (84-) ↔ (87+)
↕ ↕ ↕
(57+) ↔ (68-) (75+) ↔ (79-) (shown in shaded squares)

But now, square (57) has the same polarity as square (87), so the positive polarity is false, & the only polarity possible is the negative polarity. Therefore all squares with positive polarity must have their 9-candidates removed. This causes the 9-candidates with negative polarity to become the established values for their squares:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					9		9	9	9
row2									
row3					9	9		9	9
row4		9			9				
row5			9		9	9	~~9+~~		
row6				~~9+~~				9	
row7									9
row8				9			~~9+~~		
row9		9	9						

problem 62 continued on next page

Polarity Solution No. 62, continued.

After removing the 9-candidates from all the squares in the neighborhoods of these three newly established established values, we note that the 9-candidate in square (17) is now the only 9-candidate, so it is promoted to be the established value for that square, which than eliminates the 9-candidate from square (15):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					~~9~~		9		
row2									
row3					9	9			
row4		9			9				
row5			9		9	9			
row6								9	
row7									9
row8				9					
row9		9	9						

problem 62 continued on next page

Polarity Solution No. 62, continued.

Omitting the printing of the established 9's, we then have the final result for the 9-pattern, which is then easily analyzed for subpatterns:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3					9_{3}	9_{12}			
row4		9_{23}			9_{1}				
row5			9_{1}		9_{2}	9_{3}			
row6									
row7									
row8									
row9		9_{1}	9_{23}						

Polarity Solution No. 63

Chain: (64+) ↔ (84-) ↔ (87+) ↔ (87+) (shown in shaded squares)

Square (69) can "see" both (64+) & (79-), so its 9-candidate should be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			7						7
row3	~~7~~							7-	7
row4									
row5									
row6									
row7	7+								7-
row8	7		7	7				7+	
row9	7		7	7					

Polarity Solution No. 63, continued

After removing the 7-candidate from square (31), the remaining 7-candidate in square (23), being the only remaining 7-candidate in box A, is promoted to the established value for square (23), which then eliminates the 7-candidates from all the squares in its neighborhood, including squares (29), (83) & (93). This reduces the pattern to the following, which is immediately accessible to subpattern analyses:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			**7**						
row3								7	7
row4									
row5									
row6									
row7	7								7
row8	7			7				7	
row9	7			7					

Note that the large 7 is the established value for square (23), and is only shown in order to make the elimination of 7-candidates from squares (29), (83) & (93) more visibly clear. Established values are seldom shown in patterns, because two different sudokus may have the same subpattern while having the established values in differing squares.

Polarity Solution No. 64

Chain: (43+) ↔ (49-) ↔ (39+)
↕ ↕
(51-) ↔ (58+) (shown in shaded squares)

Square (31) can "see" both (39+) & (51-), so its 7-candidate should be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			7				7	7	
row2									
row3	~~7~~	7							7+
row4			7+						7-
row5	7-							7+	
row6									
row7	7	7						7	
row8									
row9		7	7				7	7	

continued on next page

Polarity Solution No. 64, continued.

After removing the 7-candidate from square (31), the polarity chain is extended:

Chain: (43+) ↔ (49-) ↔ (39+) ↔ (32-) ↔ (13+)
↕ ↕
(51-) ↔ (58+) (shown in shaded squares)

But now, two squares, (43+) & (13+) with the same +polarity can "see" each other, so all the squares with plus polarity must be false, and their 7-candidates must be removed. The resultant situation is shown on the next page.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			7+				7	7	
row2									
row3		7-							7+
row4			7+						7-
row5	7-							7+	
row6									
row7	7	7						7	
row8									
row9		7	7				7	7	

Polarity Solution No. 64, continued.

In the diagram below, the 7-candidates have been removed from all the squares with plus polarity, leaving only the squares with negative polarity, which must be the correct squares for the 7-candidates. We now eliminate all the 7-candidates in the neighborhoods of the squares with negative polarity. (The result of this is shown on the next page.)

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1							7	7	
row2									
row3		7-							
row4									7-
row5	7-								
row6									
row7	~~7~~	~~7~~						7	
row8									
row9		~~7~~	7				7	7	

continued on the next page

Polarity Solution No. 64, continued.

The 7-candidate in square (93) is the only one left in box G, so it must be promoted to the established value for its square, which then eliminates all 7-candidates in its neighborhood – i.e. squares (97) & (98). This action causes the 7-candidate in square (78) to become the only 7-candidate in box I, so it eliminates the 7-candidate from square (18)

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1							7	~~7~~	
row2									
row3		7							
row4									7
row5	7								
row6									
row7								7	
row8									
row9			7				~~7~~	~~7~~	

continued on the next page

Polarity Solution No. 64, continued.

The following is the resultant pattern, whose members have all been shown in large font to show that they have been promoted to the established values for their squares:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1							**7**		
row2									
row3		**7**							
row4									**7**
row5	**7**								
row6									
row7								**7**	
row8									
row9			**7**						

Polarity Solution No. 65.

Chain: (51-) ↔ (43+) ↔ (47-) ↔ (67+) ↔ (64-)
↕
(73-) ↔ (76+) (shown in shaded squares)

Square (56) can "see" both (76+) & (64-), so its 6-candidate should be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4			**6+**				**6-**		
row5	**6-**			**6**		~~**6**~~			
row6				**6-**			**6+**		
row7			**6-**			**6+**			
row8	**6**							**6**	
row9	**6**			**6**		**6**		**6**	

continued on next page

Polarity Solution No. 65, continued.

After removing the 6-candidate from square (56), the chain may be extended:

Chain: (54+) ↔ (51-) ↔ (43+) ↔ (47-) ↔ (67+) ↔ (64-)
↕
(73-) ↔ (76+) ↔ (96-) (shown in shaded squares)

Square (94) can "see" both (54+) & (64-), so its 6-candidate should be removed. [It can also "see" both (76+) & (96-)]

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2									
row3									
row4			**6+**				**6-**		
row5	**6-**			**6+**					
row6				**6-**			**6+**		
row7			**6-**			**6+**			
row8	**6**							**6**	
row9	**6**			~~**6**~~		**6-**		**6**	

Polarity Solution No. 66

Chain: (23+) ↔ (26-) ↔ (96+) ↔ (94-)
↕ ↕
(35+) ↔ (45-) ↔ (54+) ↔ (53-) (shown in shaded squares)

Squares (33) & (83) can "see" both (33+) & (83-), so their 2-candidates should be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			2+			2-			
row3	2	2	~~2~~		2+				
row4	2	2			2-				
row5			2-	2+					
row6									
row7									
row8		2	~~2~~						
row9				2-		2+			

Polarity Solution No. 66

After removing the 2-candidate from square (83), the 2-candidate in square (82) becomes unique, and cancels the 2-candidates from squares (32) & (42). The chain may be extended, but no more candidates can be erased. The resulting 2-pattern has 2 subpatterns, one for the + squares, a second for the\ - squares.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			**2+**			**2-**			
row3	**2+**				**2+**				
row4	**2-**				**2-**				
row5			**2-**	**2+**					
row6									
row7									
row8		**2**							
row9				**2-**		**2+**			

Polarity Solution No. 67

Chain: (15+)↔(19-)↔(99+)↔(78-)
↕
(27-)↔(67-) (shown in shaded squares)

Square (75) can "see" both (15+) & (78-), so its 4-candidate should be removed. Square (65) can "see" both (15+) & (67-), so its 4-candidate should also be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					4+				4-
row2				4			4+		
row3									
row4			4		4			4	
row5			4	4		4			
row6					4		4-	4	
row7				4	4	4		4-	
row8									
row9				4					4+

continued on next page

Polarity Solution No. 67

After removing the 4-candidates from squares (65) & (75), the polarity chain may be extended to include square (68-):

Chain: (15+)↔(19-)↔(99+)↔(78-)
↕
(27+)↔(67-)↔(68+) (shown in shaded squares)

Now, square (48) can "see" both (68+) & (78-), so its 4-candidate may be removed:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					**4+**				**4-**
row2				**4**			**4+**		
row3									
row4			**4**		**4**			**4**	
row5			**4**	**4**		**4**			
row6							**4-**	**4+**	
row7				**4**		**4**		**4-**	
row8									
row9				**4**					**4+**

The chain may still be extended, but no further 4-candidates can be removed by polarity.

Polarity Solution No. 68

Chain: (46+)↔(55-)↔(57+)
↕
(35+)↔(39-) (shown in shaded squares)

Squares (17), (27) & (49) can all "see" both (57+) & (39-), so their 4-candidates may all be removed:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			4	4		4	4		
row2			4				4		4
row3					4+				4-
row4						4+	4		4
row5					4-		4+		
row6									
row7				4		4			
row8									
row9				4					4+

Polarity Solution No. 69

Chain: (56-)↔(52+)↔(32-)↔(13+)↔(17-)↔(97+)
(shown in shaded squares)

Square (96) can "see" both (56-) & (97+), so its 1-candidate may be removed:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1			1+				1-		
row2									
row3		1-							1+
row4									
row5		1+				1-			
row6			1-		1	1			
row7						1			1
row8									
row9					1	1	1+		1

Polarity Solution No. 70

Chain: (81-)↔(51+)↔(55-)↔(46+)↔(26-)
↕ ↕ ↕
(82+)↔(62-)↔(69+)↔(47-) (shown in shaded squares)

Square (29) can "see" both (26-) & (69+), so its 2-candidate may be removed. This makes row2 owned by the pair of 2's in (25) & (26), which then own box B, so they eliminate the 2-candidate in (15).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					2		2		2
row2					2	2-			2
row3									
row4						2+	2-		
row5	2+				2-				
row6		2-							2+
row7									
row8	2-	2+							
row9									

Polarity Solution No. 71

Chain: (16+)↔(96-)
↕ ↕ ↕
(25-) (91+) (shown in shaded squares)

Square (21) can "see" both (91+) & (25-), so its 7-candidate may be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1						7+			7
row2	7	7			7-		7	7	
row3	7						7		
row4									
row5									
row6								7	7
row7		7			7		7	7	7
row8		7			7		7	7	7
row9	7+					7-			

continued on next page

Polarity Solution No. 71 continued

After removing the 7-candidate from square (21), the chain above may be extended:

Chain: (16+)↔(96-)
↕ ↕
(25-)) (91+)↔(31-)↔(22+) (shown in shaded squares)

Squares (27) & (28) can both "see" both (22+) & (25-), so their 7-candidates may be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1						7+			7
row2		7+			7-		7	7	
row3	7-						7		
row4									
row5									
row6								7	7
row7		7			7		7	7	7
row8		7			7		7	7	7
row9	7+					7-			

Polarity Solution No. 72

Chain: (91-)↔(21+)↔(29-)↔(49+)
↕ ↕ ↕
(77-)↔(73+)↔(33-)↔(38+) (shown in shaded squares)

Square (47) can "see" both (49+) & (77-), so its 6-candidate may be removed. Square (98) can see both (38+) & (91-), so its 6-candidate may also be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2	6+								6-
row3			6-					6+	
row4						6	6		6+
row5						6	6		
row6									
row7			6+				6-		
row8				6			6	6	
row9	6-			6				6	

continued on next page

Polarity Solution No. 72 continued

After removing the 6-candidates from squares (47) & (98), the chain above may be extended:

Chain: (84-)↔(94+)↔(91-)↔(21+)↔(29-)↔(49+)↔(46-)
↕ ↕ ↕ ↕ ↕
(77-)↔(73+)↔(33-)↔(38+) (57-)↔(56+)
(shown in shaded squares)

Squares (57) & (77) are both negative and in the same column, therefore the negative polarities are in contradiction, so the positive polarities are the only possibilities. Therefore the 6-candidates may be removed from squares (29), (33), (46), (57), (77), (84). (88) & (91) may all be removed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2	6+								6-
row3			6-					6+	
row4						6-			6+
row5						6+	6-		
row6									
row7			6+				6-		
row8				6-			6	6-	
row9	6-			6+					

continued on next page

Polarity Solution No. 72 continued\

After removing all the 6-candidates with negative polarity, the final pattern of 6-candidates is the following:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2	**6**								
row3								**6**	
row4									**6**
row5						**6**			
row6									
row7			**6**						
row8							**6**		
row9				**6**					