

Fully Solved Problems in Subpattern Analysis

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Foreword

This textbook was begun shortly after launching my website. It followed hard upon the Subpattern Analysis Textbook. Its purpose was to provide fully solved problems, including pattern analysis and testing of all subpatterns leading up to the solution for each problem. The feeling was that the Subpattern Analysis Textbook did not offer sufficiently solved examples on the use of subpatterns, especially fully-solved examples.

The reasons for this insufficiency were the design limitations placed on the Subpattern Analysis Textbook, which were imposed by the original intention of creating a hard-copy textbook for the mass market. These reasons evaporated with the creation of a website to offer the textbook for free, but it was desirable to get the textbook out as soon as possible, and it was felt that the best way of updating the original textbook was not by rewriting it but by creating further texts whose purpose would be to augment those areas felt to be lacking. By such means, new ideas for graphics and display could be more easily tried out.

At the very outset, a number of texts already existed as pieces of a previous, massive 3,000+page textbook which had been abandoned some years ago. These additional texts were placed on the website along with the Subpattern Analysis Textbook in order to assist sudoku fans in upgrading those areas subsidiary to subpattern analysis, such as group separation and polarity. It is the intention of the author to continue in this vein, and this textbook is an example of this intention. A similar effort is expected in the near future on the subject of pattern solving all by itself (with some regard for its potential use)..

What is especially new in this newer textbook on subpattern analysis is the use of pictures of the pieces of the sudoku being updated which graphically show the step-by-step logic necessary for testing subpatterns, replacing the older method of exhaustive arithmetic logic.

Gary Aldrich, June 2014.

Table of Contents

Introduction-----	9-10
Explanation of update actions-----	11-13
Example A-----	14
Analysis of 1-pattern-----	15
Test of subpattern-1-----	16
Stage 1-----	17-20
Test of subpattern-2-----	21
Stage 1-----	22-26
Stage 2-----	27-30
Stage 3-----	31-33
Example B-----	34
Analysis of 7-pattern-----	35-36
Test of subpattern-1-----	37
Stage 1-----	38-41
Stage 2-----	42-43
Test of subpattern-2-----	44
Stage 1-----	45-48
Stage 2-----	49-51
Stage 3-----	52-54
Stage 4-----	55-57
Stage 5-----	58-60
Example C-----	61
Analysis of 2-pattern-----	62
Test of subpattern-1-----	63
Stage 1-----	64-66
Stage 2-----	67-68
Test of subpattern-2-----	69
Stage 1-----	70-72
Stage 2-----	73-75
Stage 3-----	76-78
Stage 4-----	79-82
Stage 5-----	83-85
Example D-----	86
Analysis of 2-pattern-----	87-89
Test of subpattern-1-----	90
Stage 1-----	91-93
Stage 2-----	94-97
Stage 3-----	98-100
Stage 4-----	101-104

continued on next page

Example E-----	105
Analysis of 9-pattern-----	106-110
Test of subpattern-1-----	111
Stage 1-----	112-115
Stage 2-----	116-119
Stage 3-----	120-122
Example F-----	123
Analysis of 5-pattern-----	124
Test of subpattern-1-----	125
Stage 1-----	126-128
Stage 2-----	129-131
Stage 3-----	132-134
Example G-----	135
Analysis of 4-pattern-----	136-140
Test of subpattern-1-----	141
Stage 1-----	142-145
Stage 2-----	146-149
Example H-----	150
Analysis of 1-pattern-----	151-153
Test of subpattern-1-----	154
Stage 1-----	155-157
Stage 2-----	158-161
Stage 3-----	162-165
Stage 4-----	166-168
Example I-----	169
Analysis of 4-pattern-----	170-173
Test of subpattern-1-----	174
Stage 1-----	175-177
Stage 2-----	178-180
Stage 3-----	181-184
Test of subpattern-2-----	185
Stage 1-----	186-188
Test of subpattern-3-----	189
Stage 1-----	190-192
Stage 2-----	193-195
Stage 3-----	196-199
Stage 4-----	200-202
Stage 5-----	203-205
Stage 6-----	206-207

continued on next page

Example J-----	208
Analysis of 1-pattern-----	209-212
Test of subpattern-1-----	213
Stage 1-----	214-215
Test of subpattern-2-----	216
Stage 1-----	217-218
Test of subpattern-3-----	219
Stage 1-----	220-222
Stage 2-----	223-227
Stage 3-----	228-230
Stage 4-----	231-232
Example K-----	233
Analysis of 5-pattern-----	234-238
Test of subpattern-1-----	239
Stage 1-----	240-241
Stage 2-----	242
Test of subpattern-2-----	243
Stage 1-----	244-246
Stage 2-----	247
Stage 3-----	248-249
Stage 4-----	250-252
Stage 5-----	253-255
Example L-----	256
Analysis of 4-pattern-----	257-260
Test of subpattern-1-----	261
Stage 1-----	262-264
Stage 2-----	265-266
Test of subpattern-2-----	267
Stage 1-----	268-271
Test of subpattern-3-----	272
Stage 1-----	273-275
Stage 2-----	276-277
Stage 3-----	278-279
Stage 4-----	280-282
Stage 5-----	283-285
Stage 6-----	286-290

continued on next page

Example M-----	291
Analysis of 8-pattern-----	292-295
Test of subpattern-1-----	296
Stage 1-----	297-301
Stage 2-----	302
Test of subpattern-2-----	303
Stage 1-----	304-306
Stage 2-----	307-309
Stage 3-----	310-312
Stage 4-----	313-315
Stage 5-----	316-319
Example N-----	320
Analysis of 4-pattern-----	321-322
Test of subpattern-1-----	323
Stage 1-----	324-327
Stage 2-----	328-331
Stage 3-----	332-334
Test of subpattern-2-----	335-336
Removal of two failed 4-candidates-----	337
Test of resultant sudoku-----	338
Stage 1-----	339-342
Stage 2-----	343-346
Stage 3-----	347-349
Stage 4-----	350-354

end of table of contents

Introduction

It is assumed that anyone using this textbook will already have familiarized him- or herself with the terminology and methods of the Subpattern Analysis Textbook. It is hoped that the pictorial representation of the rows, columns and boxes, shown as separate pieces in this new textbook, will put the reader more in touch with the temporary updating necessary for supattern testing. We shall digress at this point to briefly mention the salient points of subpattern testing, which involves a process called "temporary updating."

When testing a subpattern with a temporary update, there is no updating of solved squares, since they are already fully solved, so the updating is made only of the blank squares, which have lists of candidates at their tops. At the end of any temporary updating, if the update has led to a contradiction, all the updates are erased, and the sudoku reverts to its original form, ready for the next temporary update by the next subpattern to be tested.

In a temporary update, new and changed values are always placed at the centers of squares; the tops of the squares being left pristine, so when the update fails, it may be totally erased, simply by erasing the middles of all squares with candidates at their tops. Those with no candidates at their tops are never changed, since they already possess solved values and are not affected by updates of any kind.

When a "blank" square is given a new, solved value, that value is placed in the middle of the square, where it will be erased at the end of the temporary update.

When the candidate list of a square is updated, the changed list of candidates is first placed at the center of the square, and it is that list in the center which is then updated. When the candidate list is to be updated, and a list of candidates already exists in the middle of the square, then it is *that* list in the center which is updated.

In this fashion, the candidate list at the top of a square is never altered during a temporary update, although it may play a part in the logic of the update when the center of its square is blank. In this fashion, at the end of a temporary update, when that update has failed and the digits at the centers of squares are erased for squares with candidates at their tops, the sudoku has not been altered, and may undergo another temporary update.

During the update, it is only those candidates in the middle of a square which are changed. The candidate list at the top is only valid when the center of the square is blank, and in such a circumstance, when changes must be made to the list of candidates for a square with nothing in its middle, a copy of the candidate list is first placed in the center of the square, and it is that copy which is affected by the operation.

When the candidate list in the middle of the square is reduced to a single candidate, that candidate is written large in the middle of the square, so that it resembles a solved square, and is treated as such during the remainder of the temporary update.

What is to be gleaned from a perusal of this textbook is a better acquaintance with the processes of temporary updating. It should be helpful just to go over all the separate updates of the row, column, and box pieces which are shown pictorially, in order to feel more confident with how these updates are arrived at.

It is probably helpful for the student to parallel the text for at least one problem. It may be tedious, but there are certain points where techniques need to be strictly followed. This should be welcomed by an experienced problem solver, who knows the importance of seeing examples.

When a temporary update is being carried out in a real-life situation, there are no pictures of sudoku pieces – just the sudoku itself, and the separation of pieces is dealt with in a much quicker and immediate visual fashion. An update taking twenty pages of pictures can frequently be carried out in ten minutes or less. No problem in this text took me more than an hour to complete, from start to finish, and several took as little as fifteen minutes. If this text suggests that the method is exhausting, it should be pointed out that it is the visual display of steps which is truly exhausting. It took me most of an afternoon for me just to complete the assembly of a single one of these problems in my word processor, while no one of them exceeded an hour to actually solve for myself in advance. Careful practice should become invisible, in that the background of action on the sudoku board is directed towards the matter at hand, and provides moments of real clarity

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Explanation of Update Actions

The only indications of update actions are the use of digits underscored and in bold. When an integer or a list of integers is in bold, it is either the instrument of change or the item which has been changed, and when it is underlined, it is the instrument of change, so instruments of change are both underlined and in bold, and those which are only changed are in bold, but not underlined. Items are often both the result of change in one step and the instrument of change in the next step. In such cases, they are in bold for having been changed in the prior step, and underlined because they are the cause of change in the next step. In addition, two or more change actions may be made in a single step. It is best to illustrate this process with some examples.

-----BOX A-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	5	<u>2</u>	9	5	2	9	5	2	9
row2	78	48	3	<u>78</u>	48	3	78	4	3
row3	1278	6	178	1278	6	<u>178</u>	1278 178	6	178

At the start, square (12) has an underlined 2 in bold. Since this is the first step, the only possible interpretation is that the 2 is the agent of change. We can see its immediate target is square (31). The first action is that the 2 removes its counterpart in square (31), leaving it with 178, as we see in the next box. Then we see two other boxes, both with all candidates underlined and in bold. The 3 squares contain the 3 digits, 1, 7 & 8. Therefore they own them, so we can remove the 8 from square (22), reducing it to a 4. The 4 is in bold, and there is no further action since the 4 appears in the last box, in bold to indicate that it has been changed.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	1278	128		3	178		9	6	12
	<u>2</u>		4	3		9	6	<u>1</u>	5
STEP 1	1278	128		3	178		9	6	12
	2	<u>8</u>	4	3	<u>78</u>		9	6	1
STEP 2	1278	128		3	178		9	6	12
	2	8	4	3	7		9	6	1

In this second example, the 1 & 2 are both underlined and in bold. Therefore they are agents of change. We notice that they can be removed from square (82), leaving an 8, which is in bold to indicate that it is the result of the removal of the other two digits. It is also underlined, so it must be an agent of change during the next step. We see that square (85) has an 8 to be deleted, which we can see has been accomplished during the next step. Square (85) has been reduced to a 7. It is in bold to show it has been changed.

-----BOX H-----									
start			step 1			step 2			
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row7	1249	1245	45	1249	1245	45	1249	1245	45
				129	125	<u>5</u>	129	12	5
row8	6	145		6	145		6	145	
		<u>4</u>	7		4	7		4	7
row9	239		35	239		35	239		35
		8			8			8	<u>3</u>

step 3				
	col 4	col 5	col 6	
row7	1249	1245	45	
	129	12	5	
row8	6	145		
		4	7	
row9	239		35	
	29	8	3	

In the start box, the 4 in square (85) is underlined and in bold. Since this is the first box, there is only one interpretation – that it is an agent of change. We note its elimination from squares (74), (75) and (76) in step 1. Squares (74) & (75) are only in bold, showing they are only the recipients of the removal action of the 4. Square (76), however, is not only in bold, but underlined, showing it to be not only a changed item, but one responsible for change by itself. In step 2, we see that it has been used to eliminate itself from square (96), leaving (96) with a 3, which is not only in bold do show it to be changed, but also underlined to show it to be an agent of change. We see in step 3 that it has eliminated itself from square (94), leaving the digits 29, in bold to show that their square has been reduced.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	16 <u>6</u>	9	2	1348	58	348	7	456	56 <u>5</u>
STEP 1	16 6	9	2	1348	58 <u>8</u>	348	7	456 <u>4</u>	56 5
STEP 2	16 6	9	2	1348 13	58 8	348 <u>3</u>	7	456 4	56 5
STEP 3	16 6	9	2	1348 1	58 8	348 3	7	456 4	56 5

5 and 6 are both underlined and in bold in the start row, indicating they are agents of change. Both may be eliminated from square (38), leaving a 4,, and 5 may be eliminated from square (35). In both cases, the 4 and 8 are underlined and in bold, indicating they are both agents of change, both reducing square (34) to the digits 1 & 3, and both reducing square (36) to a 3, with square (34) in bold, showing it to have been changed. But the other result, in square (36), is also underlined, showing it to be an agent of change in reducing square (34) to a 1, shown in bold because it has been changed.

Example A:

The following sudoku has not been subjected to any solving techniques beyond annotation and simple reduction. No effort was made to check for n-wings. It seemed easy enough to tackle immediately with a simple subpattern..

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7	56	9	2	58	1	3	68	4
row2	456	3	12	59	589	46	7	12	68
row3	8	46	12	7	46	3	29	129	5
row4	2	568	4	39	39	68	1	56	7
row5	56	9	58	1	7	2468	26	23456	36
row6	1	7	3	45	456	246	8	2456	9
row7	9	28	7	6	23	5	4	38	1
row8	45	12458	58	34	1234	7	69	3689	368
row9	3	14	6	8	14	9	5	7	2

	1	
1	1	1
		1

	2	
2		
		2

3	3	3
3		
3		

		4
4		
		4

		5
	5	

	6	

7	7	7
7	7	7
7	7	7

8		
		8
	8	

9		
9		9
9	9	

This is the 1-pattern. Its analysis is too easy to bother to formalize. There are two independent subpatterns. Looking first at the one in boxes A and C, we note that the two subpatterns 1_A and 1_B are mutually exclusive. One must be true and the other false. Still, neither one can be shown to be either. Try them out.

The subpattern in boxes G and H has the same attributes. Again, it is clear that one is true and the other false. We'll try subpattern 1_1 first.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2			1_A					1_B	
row3			1_B					1_A	
row4									
row5									
row6									
row7									
row8		1_1			1_2				
row9		1_2			1_1				

We'll first test subpattern 1 in squares (82) and (95):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7	56	9	2	58	1	3	68	4
row2	456	3	12	59	589	46	7	12	68
row3	8	46	12	7	46	3	29	129	5
row4	2	568	4	39	39	68	1	56	7
row5	56	9	58	1	7	2468	26	23456	36
row6	1	7	3	45	456	246	8	2456	9
row7	9	28	7	6	23	5	4	38	1
row8	45	12458 1	58	34	1234	7	69	3689	368
row9	3	14	6	8	14 1	9	5	7	2

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

COLUMN 2						
	start	step1	step2	step3	step4	step5
row1	56	56	56	56 <u>5</u>	56 5	56 5
row2	3	3	3	3	3	3
row3	46	46	46 <u>6</u>	46 6	46 6	46 6
row4	568	568	568	568 58	568 <u>8</u>	568 8
row5	9	9	9	9	9	9
row6	7	7	7	7	7	7
row7	28	28	28	28	28	28 2
row8	12458 <u>1</u>	12458 1	12458 1	12458 1	12458 1	12458 1
row9	14	14 <u>4</u>	14 4	14 4	14 4	14 4

-----BOX G-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	9	28 2	7	9	28 2	7	9	28 2	7
row2	45	12458 1	58	45 <u>5</u>	12458 1	58	45 5	12458 1	58 8
row3	3	14 <u>4</u>	6	3	14 4	6	3	14 4	6

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	9	²⁸ <u>2</u>	7	6	²³	5	4	³⁸	1
STEP 1	9	²⁸ 2	7	6	²³ <u>3</u>	5	4	³⁸	1
STEP 2	9	²⁸ 2	7	6	²³ 3	5	4	³⁸ 8	1

	COLUMN 8			
	start	step1	step2	step3
row1	68	68 <u>6</u>	68 6	68 6
row2	12	12	12	12
row3	129	129	129	129
row4	56	56	56 <u>5</u>	56 5
row5	23456	23456	23456 2345	23456 234
row6	2456	2456	2456 245	2456 24
row7	³⁸ <u>8</u>	³⁸ 8	³⁸ 8	³⁸ 8
row8	3689	3689 369	3689 39	3689 39
row9	7	7	7	7

-----BOX H-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	6	23 <u>3</u>	5	6	23 3	5	6	23 3	5
row2	34	1234	7	34	1234 24	7	34	1234	7
row3	8	14 <u>1</u>	9	8	14 1	9	8	14 1	9

-----BOX D-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	2	568 <u>8</u>	4	2	568 8	4	2	568 8	4
row2	56	9	58	56	9	58 <u>5</u>	56	9	58 5
row3	1	7	3	1	7	3	1	7	3

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	56 <u>6</u>	9	58 5	1	7	2468	26	23456 234	36
STEP 1	56 6	9	58 5	1	7	2468 248	26 <u>2</u>	23456 234	36 <u>3</u>
STEP 2	56 6	9	58 5	1	7	2468 48	26 2	23456 <u>4</u>	36 3
STEP 3	56 6	9	58 5	1	7	2468 8	26 2	23456 4	36 3

This is the sudoku after updating col 2, box G, row 7, col 8, box H, box D and row 9:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7	56	9	2	58	1	3	68	4
row2	456	3	12	59	589	46	7	12	68
row3	8	46	12	7	46	3	29	129	5
row4	2	568	4	39	39	68	1	56	7
row5	56	9	58	1	7	2468	26	23456	36
row6	1	7	3	45	456	246	8	2456	9
row7	9	28	7	6	23	5	4	38	1
row8	45	12458	58	34	1234	7	69	3689	368
row9	3	14	6	8	14	9	5	7	2

Look at box F. The three squares (57), (58) and (68) are composed of the two digits 2 and 4. This is a violation of 9-completeness. Therefore subpattern-1 is false, which means that subpattern-2 is the correct subpattern. We still need to evaluate subpattern-2 in order to arrive at the solution to the sudoku.

Evaluating subpattern-2 in squares (85) and (92):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7	56	9	2	58	1	3	68	4
row2	456	3	12	59	589	46	7	12	68
row3	8	46	12	7	46	3	29	129	5
row4	2	568	4	39	39	68	1	56	7
row5	56	9	58	1	7	2468	26	23456	36
row6	1	7	3	45	456	246	8	2456	9
row7	9	28	7	6	23	5	4	38	1
row8	45	12458	58	34	1234	7	69	3689	368
row9	3	14	6	8	14	9	5	7	2

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX H-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	6	23	5	6	23	5	6	23	5
row2	34	1234	7	34	1234	7	34	1234	7
		<u>1</u>			1		<u>3</u>	1	
row3	8	14	9	8	14	9	8	14	9
					<u>4</u>			4	

	step 3		
	col 1	col 2	col 3
row1	6	23	5
		2	
row2	34	1234	7
	3	1	
row3	8	14	9
		4	

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	9	28	7	6	23	5	4	38	1
		<u>2</u>							
STEP 1	9	28	7	6	23	5	4	38	1
		8			2				
STEP 2	9	28	7	6	23	5	4	38	1
		8			2			3	

-----BOX G-----

start

	col 1	col 2	col 3
row1	9	28 <u>8</u>	7
row2	45	12458	58
row3	3	14 <u>1</u>	6

step 1

	col 1	col 2	col 3
row1	9	28 8	7
row2	45	12458 245	58 <u>5</u>
row3	3	14 1	6

step 2

	col 1	col 2	col 3
row1	9	28 8	7
row2	45 4	12458 24	58 5
row3	3	14 1	6

step 3

	col 1	col 2	col 3
row1	9	28 8	7
row2	45 4	12458 2	58 5
row3	3	14 1	6

COLUMN 4				
	start	step1	step2	step3
row1	2	2	2	2
row2	59	59	59	59
			<u>5</u>	5
row3	7	7	7	7
row4	39	39	39	39
		<u>9</u>	9	9
row5	1	1	1	1
row6	45	45	45	45
				4
row7	6	6	6	6
row8	34	34	34	34
	<u>3</u>	3	3	3
row9	8	8	8	8

COLUMN 5						
	start	step1	step2	step3	step4	step5
row1	58	58	58	58 <u>8</u>	58 8	58 8
	589	589	589	589 89	589 <u>9</u>	589 9
row2	46	46 <u>6</u>	46 6	46 6	46 6	46 6
	39	39	39	39	39	39 3
row3	7	7	7	7	7	7
	456	456 56	456 <u>5</u>	456 5	456 5	456 5
row4	23	23	23	23	23	23
	2	2	2	2	2	2
row5	1234	1234	1234	1234	1234	1234
	1	1	1	1	1	1
row6	14	14	14	14	14	14
	<u>4</u>	4	4	4	4	4

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	7	56	9	2	58 <u>8</u>	1	3	68	4
STEP 1	7	56	9	2	58 8	1	3	68 <u>6</u>	4
STEP 2	7	56 5	9	2	58 8	1	3	68 6	4

After updates of box H, row 7, box G, col 4, col 5 and row 1:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7	56	9	2	58	1	3	68	4
row2	456	3	12	59	589	46	7	12	68
row3	8	46	12	7	46	3	29	129	5
row4	2	568	4	39	39	68	1	56	7
row5	56	9	58	1	7	2468	26	23456	36
row6	1	7	3	45	456	246	8	2456	9
row7	9	28	7	6	23	5	4	38	1
row8	45	12458	58	34	1234	7	69	3689	368
row9	3	14	6	8	14	9	5	7	2

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	COLUMN 2		
	start	step1	step2
row1	56 <u>5</u>	56 5	56 5
row2	3	3	3
row3	46	46	46 4
row4	568	568 <u>6</u>	568 6
row5	9	9	9
row6	7	7	7
row7	28 <u>8</u>	28 8	28 8
row8	12458 2	12458 2	12458 2
row9	14 1	14 1	14 1

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4 START	2	568 <u>6</u>	4	39 9	39 3	68	1	56	7
STEP 1	2	568 6	4	39 9	39 3	68 8	1	56 5	7

-----BOX D-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	2	568 <u>6</u>	4	2	568 6	4	2	568 6	4
row2	56	9	58	56 <u>5</u>	9	58	56 5	9	58 8
row3	1	7	3	1	7	3	1	7	3

COLUMN 8									
	start		step1		step2		step3		step4
row1	68		68		68		68		68
	<u>6</u>		6		6		6		6
row2	12		12		12		12		12
					<u>1</u>		1		1
row3	129		129		129		129		129
					19		<u>9</u>		9
row4	56		56		56		56		56
	<u>5</u>		5		5		5		5
row5	23456		23456		23456		23456		23456
			<u>24</u>		24		24		24
row6	2456		2456		2456		2456		2456
			<u>24</u>		24		24		24
row7	38		38		38		38		38
	<u>3</u>		3		3		3		3
row8	3689		3689		3689		3689		3689
			89		89		89		8
row9	7		7		7		7		7

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	45 4	12458 2	58 5	34 <u>3</u>	1234 1	7	69	3689 <u>8</u>	368
STEP 1	45 4	12458 2	58 5	34 3	1234 1	7	69	3689 8	368 <u>6</u>
STEP 2	45 4	12458 2	58 5	34 3	1234 1	7	69 9	3689 8	368 6

		col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6	START	1	7	3	45 <u>4</u>	456 5	246	8	2456 24	9
	STEP 1	1	7	3	45 4	456 5	246 26	8	2456 <u>2</u>	9
	STEP 2	1	7	3	45 4	456 5	246 6	8	2456 2	9

After updates of col 2, row 4, box D, col 8, row 8 and row 6:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7	56 5	9	2	58 8	1	3	68 6	4
row2	456	3	12	59 5	589 9	46	7	12 1	68
row3	8	46 4	12	7	46 6	3	29	129 9	5
row4	2	568 6	4	39 9	39 3	68 8	1	56 5	7
row5	56 5	9	58 8	1	7	2468	26	23456 24	36
row6	1	7	3	45 4	456 5	246 6	8	2456 2	9
row7	9	28 8	7	6	23 2	5	4	38 3	1
row8	45 4	12458 2	58 5	34 3	1234 1	7	69 9	3689 8	368 6
row9	3	14 1	6	8	14 4	9	5	7	2

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX F-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	1	56 5	7	1	56 5	7	1	56 5	7
row2	26	23456 24	36	26 <u>6</u>	23456 4	36	26 6	23456 4	36 3
row3	8	2456 <u>2</u>	9	8	2456 2	9	8	2456 2	9

COLUMN 6-----

	start	step1	step2
row1	1	1	1
row2	46	46 <u>4</u>	46 4
row3	3	3	3
row4	68 <u>8</u>	68 8	68 8
row5	2468 24	2468 24	2468 2
row6	246 <u>6</u>	246 6	246 6
row7	5	5	5
row8	7	7	7
row9	9	9	9

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	8	46 4	12	7	46 6	3	29	129 <u>9</u>	5
STEP 1	8	46 4	12	7	46 6	3	29 <u>2</u>	129 9	5
STEP 2	8	46 4	12 1	7	46 6	3	29 2	129 9	5
	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	456	3	12	59 <u>5</u>	589 9	46 <u>4</u>	7	12 <u>1</u>	68
STEP 1	456 <u>6</u>	3	12 2	59 5	589 9	46 4	7	12 1	68
STEP 2	456 6	3	12 2	59 5	589 9	46 4	7	12 1	68 8

After updates of box F, col 6, row 3 and row 2:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7	56	9	2	58	1	3	68	4
row2	456	3	12	59	589	46	7	12	68
row3	8	46	12	7	46	3	29	129	5
row4	2	568	4	39	39	68	1	56	7
row5	56	9	58	1	7	2468	26	23456	36
row6	1	7	3	45	456	246	8	2456	9
row7	9	28	7	6	23	5	4	38	1
row8	45	12458	58	34	1234	7	69	3689	368
row9	3	14	6	8	14	9	5	7	2

A check of all rows, columns and boxes shows them all to be 9-perfect, so this is the solution.

An interesting result is that 1_B is implied by 1_2 .

Example B.

The following sudoku had been through trapping, annotation and n-wings up to the 6-pattern. The 7-pattern was so simple I decided to try it out for subpattern analysis.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	13789	189	2	35	58	4	137	135	6
row2	134	6	13	235	7	9	123	8	1345
row3	5	47	38	1	268	368	9	23	47
row4	6	12579	159	8	25	37	4	12359	1359
row5	18	3	158	4	9	26	26	7	15
row6	79	259	4	357	256	1	36	23569	8
row7	13489	1489	7	6	14	5	138	139	2
row8	189	1589	15689	27	3	28	1678	4	19
row9	2	148	1368	9	14	78	5	136	137

1	1	2	3	4	5
1			3	4	
		2	3	4	5
6	6		7	8	9
6			7	8	9
	6				9

This is the 7-pattern. What makes it so desirable is the number of boxes it appears in, plus the fact that it is a bi-pattern, and is therefore extremely easy to analyze. We begin subpattern-1 with the 7 in square (11), with the box order A→C→I→H→E→D.

A		C
D	E	
	H	I

→		↓
•	←	
	↑	←

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7 ₁	→	→	→	→	→	7		
row2					X				
row3		7							7 ₁
row4		7 ₁				7			↓
row5						↑		X	↓
row6	7	←	←	7 ₁		↑			↓
row7			X			↑			↓
row8				7	←	←	7 ₁		↓
row9						7 ₁			7

As we go from the preceding box to the following box, there are two 7-candidates in the following box, one of which is in the same row or column of the subpattern member in the preceding box, leaving only one possible candidate to choose in the following box. The calculation of subpattern-1 is shown above. Subpattern-2 is calculated in the same manner. The resulting subpattern analysis is shown on the next page.

This is the completed analysis of the 7-pattern..

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7 ₁						7 ₂		
row2					X				
row3		7 ₂							7 ₁
row4		7 ₁				7 ₂			
row5								X	
row6	7 ₂			7 ₁					
row7			X						
row8				7 ₂			7 ₁		
row9						7 ₁			7 ₂

We'll first test subpattern-1, with 7-candidates in squares (11), (39), (87), (96), (64) and (42):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	13789 7	189	2	35	58	4	137	135	6
row2	134	6	13	235	7	9	123	8	1345
row3	5	47	38	1	268	368	9	23	47 7
row4	6	12579 7	159	8	25	37	4	12359	1359
row5	18	3	158	4	9	26	26	7	15
row6	79	259	4	357 7	256	1	36	23569	8
row7	13489	1489	7	6	14	5	138	139	2
row8	189	1589	15689	27	3	28	1678 7	4	19
row9	2	148	1368	9	14	78 7	5	136	137

solved squares in black

	1	2	3	4	5	6	7	8	9
1	■	■	■	■	■	■	■	■	■
2	■	■	■	■	■	■	■	■	■
3	■	■	■	■	■	■	■	■	■
4	■	■	■	■	■	■	■	■	■
5	■	■	■	■	■	■	■	■	■
6	■	■	■	■	■	■	■	■	■
7	■	■	■	■	■	■	■	■	■
8	■	■	■	■	■	■	■	■	■
9	■	■	■	■	■	■	■	■	■

-----BOX A-----

start

	col 1	col 2	col 3
row1	13789 <u>7</u>	189	2
row2	134	6	13
row3	5	47	38

step 1

	col 1	col 2	col 3
row1	13789 7	189	2
row2	134	6	13
row3	5	47 <u>4</u>	38

step 2

	col 1	col 2	col 3
row1	13789 7	189	2
row2	134 <u>13</u>	6	<u>13</u>
row3	5	47 4	38

step 3

	col 1	col 2	col 3
row1	13789 7	189 89	2
row2	134 13	6	
row3	5	47 4	<u>8</u>

step 4

	col 1	col 2	col 3
row1	13789 7	189 9	2
row2	134 13	6	
row3	5	47 4	8

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	134 <u>13</u>	6	<u>13</u>	235	7	9	123	8	1345
STEP 1	134 13	6	13	235 25	7	9	123 <u>2</u>	8	1345 45
STEP 2	134 13	6	13	235 5	7	9	123 2	8	1345 4

-----BOX C-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	137	135	6	137	135	6	137	135	6
				13			<u>1</u>	15	
row2	123		1345	123		1345	123		1345
	<u>2</u>	8	4	2	8	4	2	8	4
row3		23	47		23	47		23	47
	9		<u>7</u>	9	<u>3</u>	7	9	3	7

	step 3		
	col 1	col 2	col 3
row1	137	135	
	1	5	6
row2	123		1345
	2	8	4
row3		23	47
	9	3	7

-----BOX B-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	35	58	4	35	58	4	35	58	4
				<u>3</u>	<u>8</u>		3	8	
row2	235			235			235		
	<u>5</u>	7	9	5	7	9	5	7	9
row3		268	368		268	368		268	368
	1			1			1	26	<u>6</u>

	step 3		
	col 1	col 2	col 3
row1	35	58	
	3	8	4
row2	235		
	5	7	9
row3		268	368
	1	2	6

COLUMN 6			
	start	step1	step2
row1	4	4	4
row2	9	9	9
row3	368 <u>6</u>	368 6	368 6
row4	37	37 3	37 3
row5	26	26 <u>2</u>	26 2
row6	1	1	1
row7	5	5	5
row8	28	28	28 8
row9	78 <u>7</u>	78 7	78 7

-----BOX E-----									
	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	8	25	37 3	8	<u>5</u>	37 3	8	5	37 3
row2	4	9	26 <u>2</u>	4	9	26 2	4	9	26 2
row3	357 7	256	1	357 7	256 56	1	357 7	256 6	1

After updating box A, row 2, box C, box B, col 6 and box E:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	13789 7	189 9	2	35 3	58 8	4	137 1	135 5	6
row2	134 13	6	13	235 5	7	9	123 2	8	1345 4
row3	5	47 4	38 8	1	268 2	368 6	9	23 3	47 7
row4	6	12579 7	159	8	25 5	37 3	4	12359	1359
row5	18	3	158	4	9	26 2	26	7	15
row6	79	259	4	357 7	256 6	1	36	23569	8
row7	13489	1489	7	6	14	5	138	139	2
row8	189	1589	15689	27	3	28 8	1678 7	4	19
row9	2	148	1368	9	14	78 7	5	136	137

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	79	259	4	<u>7</u>	<u>6</u>	1	36	23569	8
STEP 1	79 <u>9</u>	259	4	357 7	256 6	1	36 <u>3</u>	23569 2359	8
STEP 2	79 9	259 25	4	357 7	256 6	1	36 3	23569 25	8

	COLUMN 8			
	start	step1	step2	step3
row1	135 <u>5</u>	135 5	135 5	135 5
row2	8	8	8	8
row3	23 <u>3</u>	23 3	23 3	23 3
row4	12359	12359 129	12359 <u>19</u>	12359 19
row5	7	7	7	7
row6	23569 25	23569 <u>2</u>	23569 2	23569 2
row7	139	139 19	139 <u>19</u>	139 19
row8	4	4	4	4
row9	136	136 16	136 16	136 6

-----BOX I-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	138	139 <u>19</u>	2	138 38	139 19	2	138 8	139 19	2
row2	1678 <u>7</u>	4	<u>19</u>	1678 7	4	19	1678 7	4	19
row3	5	136 6	137	5	136 6	137 <u>3</u>	5	136 6	137 3

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	79	259		357	256		36	23569	
	9	25	4	7	6	1	3	<u>2</u>	8
STEP 1	79	259		357	256		36	23569	
	9	5	4	7	6	1	3	2	8

-----BOX D-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	6	12579 7	159	6	12579 7	159 <u>1</u>	6	12579 7	159 1
row2	18	3	158	18	3	158 18	18 8	3	158 8
row3	79 <u>9</u>	259 <u>5</u>	4	79 9	259 5	4	79 9	259 5	4

**But now we have a contradiction in box D, with the duplicate 8's.
Therefore subpattern-1 is invalid.**

By the failure of subpattern-1, we know that subpattern-2 is valid, but we still need to update the sudoku with subpattern-2 in order to arrive at the solution. Subpattern-2 has 7's in squares 917), (32), (46), (61), (84), and (99).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	13789	189	2	35	58	4	137	135	6
row2	134	6	13	235	7	9	123	8	1345
row3	5	47	38	1	268	368	9	23	47
row4	6	12579	159	8	25	37	4	12359	1359
row5	18	3	158	4	9	26	26	7	15
row6	79	259	4	357	256	1	36	23569	8
row7	13489	1489	7	6	14	5	138	139	2
row8	189	1589	15689	27	3	28	1678	4	19
row9	2	148	1368	9	14	78	5	136	137

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

COLUMN 6					
	start	step1	step2	step3	step4
row1	4	4	4	4	4
row2	9	9	9	9	9
row3	368	368	368 36	368 36	368 3
row4	37 <u>7</u>	37 7	37 7	37 7	37 7
row5	26	26	26	26 <u>6</u>	26 6
row6	1	1	1	1	1
row7	5	5	5	5	5
row8	28	28	28 <u>2</u>	28 2	28 2
row9	78	78 <u>8</u>	78 8	78 8	78 8

-----BOX B-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	35	58	4	35 <u>5</u>		4	35 5	58 <u>8</u>	4
row2	235	7	9	235 25	7	9	235 <u>2</u>	7	9
row3	1	268	368 <u>3</u>	1	268	368 3	1	268	368 3

BOX B continued on next page

BOX B continued

step 3

	col 1	col 2	col 3
row1	35 5	58 8	4
row2	235 2	7	9
row3	1	268 6	368 3

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	5	47 <u>7</u>	38	1	268 6	368 <u>3</u>	9	23	47

STEP 1	5	47 7	38 8	1	268 6	368 3	9	23 2	47 4
--------	---	----------------	----------------	---	-----------------	-----------------	---	----------------	----------------

COLUMN 4

	start	step1
row1	35 <u>5</u>	35 5
row2	235 2	235 2
row3	1	1
row4	8	8
row5	4	4
row6	357 3	357 3
row7	6	6
row8	27 <u>7</u>	27 7
row9	9	9

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	134	6	13	235 <u>2</u>	7	9	123	8	1345
STEP 1	134	6	<u>13</u>	235 2	7	9	123 <u>13</u>	8	1345
STEP 2	134 <u>4</u>	6	13	235 2	7	9	123 13	8	1345 45
STEP 3	134 4	6	13	235 2	7	9	123 13	8	1345 5

	COLUMN 9			
	start	step1	step2	step3
row1	6	6	6	6
	1345	1345	1345	1345
row2	<u>5</u>	5	5	5
	47	47	47	47
row3	4	4	4	4
	1359	1359	1359	1359
row4		139	39	3
	15	15	15	15
row5		<u>1</u>	1	1
row6	8	8	8	8
row7	2	2	2	2
	19	19	19	19
row8			<u>9</u>	9
	137	137	137	137
row9	7	7	7	7

After updates of col 6,k box B, row 3, col 4, row 2, and col 9:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	13789	189	2	35	58	4	137	135	6
row2	134 4	6	13	235 2	7	9	123 13	8	1345 5
row3	5	47 7	38 8	1	268 6	368 3	9	23 2	47 4
row4	6	12579	159	8	25	37 7	4	12359	1359 3
row5	18	3	158	4	9	26 6	26	7	15 1
row6	79 7	259	4	357 3	256	1	36	23569	8
row7	13489	1489	7	6	14	5	138	139	2
row8	189	1589	15689	27 7	3	28 2	1678	4	19 9
row9	2	148	1368	9	14	78 8	5	136	137 7

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	18	3	158	4	9	<u>6</u>	26	7	<u>1</u>

STEP 1	18	3	158	4	9	6	26	7	1
	<u>8</u>						2		

STEP 2	18	3	158	4	9	6	26	7	1
	8		5				2		

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	13789	189	2	<u>5</u>	<u>8</u>	4	<u>7</u>	135	6

STEP 1	13789	189	2	5	8	4	137	135	6
	<u>139</u>	<u>19</u>					7	<u>13</u>	

COLUMN 1

	start	step1
row1	13789 139	13789 139
row2	134 <u>4</u>	134 4
row3	5	5
row4	6	6
row5	18 <u>8</u>	18 8
row6	79 7	79 7
row7	13489	13489 <u>139</u>
row8	189	189 <u>19</u>
row9	2	2

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	189 19	1589	15689	27 <u>7</u>	3	28 2	1678	4	19 <u>9</u>
STEP 1	189 <u>1</u>	1589 158	15689 1568	27 7	3	28 2	1678 168	4	19 9
STEP 2	189 1	1589 58	15689 568	27 7	3	28 2	1678 68	4	19 9

	COLUMN 3			
	start	step1	step2	step3
row1	2	2	2	2
row2	13	13	<u>13</u>	13
row3	38 <u>8</u>	38 8	38 8	38 8
row4	159	159 19	159 19	159 9
row5	158 <u>5</u>	158 5	158 5	158 5
row6	4	4	4	4
row7	7	7	7	7
row8	15689 568 <u>6</u>	15689 6	15689 6	15689 6
row9	1368	1368 136	1368 <u>13</u>	1368 13

After updates of row 5, row 1, col 1, row 8 and col 3:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	13789 139	189 19	2	35 5	58 8	4	137 7	135 13	6
row2	134 4	6	13	235 2	7	9	123 13	8	1345 5
row3	5	47 7	38 8	1	268 6	368 3	9	23 2	47 4
row4	6	12579	159 9	8	25	37 7	4	12359	1359 3
row5	18 8	3	158 5	4	9	26 6	26 2	7	15 1
row6	79 7	259	4	357 3	256	1	36	23569	8
row7	13489 139	1489	7	6	14	5	138	139	2
row8	189 1	1589 58	15689 6	27 7	3	28 2	1678 68	4	19 9
row9	2	148	1368 13	9	14	78 8	5	136	137 7

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX D-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	6	12579	159 <u>9</u>	6	12579 12	159 9	6	12579 1	159 9
row2	18 8	3	158 <u>5</u>	18 8	3	158 5	18 8	3	158 5
row3	79 <u>7</u>	259	4	79 7	259 <u>2</u>	4	79 7	259 2	4

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4 START	6	12579 <u>1</u>	159 <u>9</u>	8	25	37 7	4	12359	1359 <u>3</u>
STEP 1	6	12579 1	159 9	8	25	37 7	4	12359 25	1359 3

-----BOX E-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row1	8	25	37 7	8	25	37 7
row2	4	9	26 <u>6</u>	4	9	26 6
row3	357 3	256	1	357 3	256 25	1

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	79 7	259 <u>2</u>	4	357 <u>3</u>	256	1	36	23569	8
STEP 1	79 7	259 2	4	357 3	256 56	1	36 <u>6</u>	23569 569	8
STEP 2	79 7	259 2	4	357 3	256 <u>5</u>	1	36 6	23569 59	8
STEP 3	79 7	259 2	4	357 3	256 5	1	36 6	23569 <u>9</u>	8

-----BOX F-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row1	4	12359 25	1359 3	4	5	3
row2	26 <u>2</u>	7	15 1	26 2	7	15 1
row3	36 6	23569 9	8	36 6	23569 9	8

	COLUMN 8		
	start	step1	step2
row1	135 13	135 <u>13</u>	135 13
row2	8	8	8
row3	23 <u>2</u>	23 2	23 2
row4	12359 25	12359 5	12359 5
row5	7	7	7
row6	23569 <u>9</u>	23569 9	23569 9
row7	139	139 <u>13</u>	139 13
row8	4	4	4
row9	136	136	136 6

After updates of box D, row 4, box E, row 6, box F and col 8:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	13789 139	189 19	2	35 5	58 8	4	137 7	135 13	6
row2	134 4	6	13	235 2	7	9	123 13	8	1345 5
row3	5	47 7	38 8	1	268 6	368 3	9	23 2	47 4
row4	6	12579 1	159 9	8	25 7	37 4	12359 5	1359 3	
row5	18 8	3	158 5	4	9	26 6	26 2	7	15 1
row6	79 7	259 2	4	357 3	256 5	1	36 6	23569 9	8
row7	13489 139	1489	7	6	14	5	138 13	139 13	2
row8	189 1	1589 58	15689 6	27 7	3	28 2	1678 68	4	19 9
row9	2	148	1368 13	9	14	78 8	5	136 6	137 7

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX I-----									
start			step 1			step 2			
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	138	139 13	2	138	139 13	2	138 13	139 13	2
row2	1678 68	4	9	1678 <u>8</u>	4	9	1678 8	4	9
row3	5	136 <u>6</u>	137 7	5	136 6	137 7	5	136 6	137 7

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	13489 139	1489	7	6	14	5	138 <u>13</u>	139 13	2
STEP 1	13489 <u>9</u>	1489 489	7	6	14 <u>4</u>	5	138 13	139 13	2
STEP 2	13489 9	1489 8	7	6	14 4	5	138 13	139 13	2

COLUMN 2

	start	step1
row1	189 19	189 9
row2	6	6
row3	47 7	47 7
row4	12579 <u>1</u>	12579 1
row5	3	3
row6	259 2	259 2
row7	1489 <u>8</u>	1489 8
row8	1589 58	1589 5
row9	148	148 4

COLUMN 1

	start	step1
row1	13789 139	13789 3
row2	134 4	134 4
row3	5	5
row4	6	6
row5	18 8	18 8
row6	79 7	79 7
row7	13489 <u>9</u>	13489 9
row8	189 <u>1</u>	189 1
row9	2	2

-----BOXA-----

	start		
	col 1	col 2	col 3
row1	13789 <u>3</u>	189 9	2
row2	134 4	6	13
row3	5	47 7	38 8

	step 1		
	col 1	col 2	col 3
row1	13789 3	189 9	2
row2	134 4	6	1
row3	5	47 7	38 8

After updates of box I, row 7, col 2, row 9, col 1, ad boxA:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	13789 3	189 9	2	35 5	58 8	4	137 7	135 13	6
row2	134 4	6	13 1	235 2	7	9	123 13	8	1345 5
row3	5	47 7	38 8	1	268 6	368 3	9	23 2	47 4
row4	6	12579 1	159 9	8	25 7	37 4	12359 5	1359 3	
row5	18 8	3	158 5	4	9	26 6	26 2	7	15 1
row6	79 7	259 2	4	357 3	256 5	1	36 6	23569 9	8
row7	13489 9	1489 8	7	6	14 4	5	138 13	139 13	2
row8	189 1	1589 5	15689 6	27 7	3	28 2	1678 8	4	19 9
row9	2	148 4	1368 13	9	14	78 8	5	136 6	137 7

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

ROW 1	START	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
		13789 <u>3</u>	189 9	2	35 <u>5</u>	58 8	4	137 7	135	6
	STEP 1	13789 3	189 9	2	35 5	58 8	4	137 7	135 1	6
ROW 2	START	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
		134 4	6	13 <u>1</u>	235 <u>2</u>	7	9	123	8	1345 5
	STEP 1	134 4	6	13 1	235 2	7	9	123 3	8	1345 5
ROW 4	START	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
		6	12579 1	159 9	8	25 2	37 7	4	12359 <u>5</u>	1359 3
	STEP 1	6	12579 1	159 9	8	25 2	37 7	4	12359 5	1359 3

COLUMN 7	
	start
row1	137 7
row2	123 <u>3</u>
row3	9
row4	4
row5	26 2
row6	36 6
row7	138 13
row8	1678 8
row9	5

step1
137 7
123 3
9
4
26 2
36 6
138 <u>1</u>
1678 8
5

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	13489 9	1489 8	7	6	14 4	5	138 <u>1</u>	139 13	2

STEP 1	13489 9	1489 8	7	6	14 4	5	138 1	139 3	2
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	2	148 <u>4</u>	1368 13	9	14	78 8	5	136 6	137 7

STEP 1	2	148 4	1368 13	9	14 <u>1</u>	78 8	5	136 6	137 7
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STEP 2	2	148 4	1368 3	9	14 1	78 8	5	136 6	137 7
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After updates of row 2, row 1, row 4, col 7, row 7 and row 9:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	13789 3	189 9	2	35 5	58 8	4	137 7	135 1	6
row2	134 4	6	13 1	235 2	7	9	123 3	8	1345 5
row3	5	47 7	38 8	1	268 6	368 3	9	23 2	47 4
row4	6	12579 1	159 9	8	25 5	37 7	4	12359 5	1359 3
row5	18 8	3	158 5	4	9	26 6	26 2	7	15 1
row6	79 7	259 2	4	357 3	256 5	1	36 6	23569 9	8
row7	13489 9	1489 8	7	6	14 4	5	138 1	139 3	2
row8	189 1	1589 5	15689 6	27 7	3	28 2	1678 8	4	19 9
row9	2	148 4	1368 3	9	14 1	78 8	5	136 6	137 7

All rows, columns and boxes are 9-perfect, so this is the solution.

Example C.

During the process of looking for n-wings, the 2-pattern looked so inviting it had to be tested, without doing any further search for n-wings.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	12	9	4	18	3	7	128	6
row2	27	8	47	579	6	157	129	3	49
row3	3	14	467	2	189	78	5	18	489
row4	9	24	467	8	125	256	3	1567	57
row5	268	5	38	369	7	126	169	4	89
row6	678	36	1	3569	359	4	69	5678	2
row7	68	36	2	3567	358	9	4	567	1
row8	1	7	358	356	4	2568	26	9	35
row9	4	9	35	1	235	567	8	2567	357

1		
1	1	1

	2	
		2
2		

3	3	3
		3

	4	
		4
4	4	4

5		5
5		

	6	6

		7
	7	
7		

8		
	8	
		8

9		
9		
9	9	0

This is the solved 2-pattern. We won't bother with a formal pattern evaluation. It's too obvious, the main principle being that two members of the same subpattern cannot be in the same row or column.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		2_1						2_2	
row2	2_2						2_1		
row3				X					
row4		2_2			2_1				
row5	2_1					2_2			
row6									X
row7			X						
row8						2_1	2_2		
row9					2_2			2_1	

We'll first test subpattern-1, with 2's in squares (12), (27), (45), (51), (86) and (98):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	12 2	9	4	18	3	7	128	6
row2	27	8	47	579	6	157	129 2	3	49
row3	3	14	467	2	189	78	5	18	489
row4	9	24	467	8	125 2	256	3	1567	57
row5	268 2	5	38	369	7	126	169	4	89
row6	678	36	1	3569	359	4	69	5678	2
row7	68	36	2	3567	358	9	4	567	1
row8	1	7	358	356	4	2568 2	26	9	35
row9	4	9	35	1	235	567	8	2567 2	357

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX A-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	5	<u>2</u>	9	5	2	9	5	2	9
row2	27	8	47	27	<u>7</u>	8	27	8	<u>4</u>
row3	3	14	467	3	14	467	3	14	467 46

	step 3		
	col 1	col 2	col 3
row1	5	2	9
row2	<u>7</u>	8	4
row3	3	1	6

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	27 <u>7</u>	8	47 <u>4</u>	579	6	157	129 2	3	49
STEP 1	27 7	8	47 4	579 59	6	157 15	129 2	3	49 <u>9</u>
STEP 2	27 7	8	47 4	579 <u>5</u>	6	157 15	129 2	3	49 9
STEP 3	27 7	8	47 4	579 5	6	157 1	129 2	3	49 9

-----BOX B-----									
start			step 1			step 2			
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	4	18	3	4	<u>8</u>	3	4	8	3
row2	579		157	579		157	579		157
	5	6	<u>1</u>	5	6	1	5	6	1
row3		189	78		189	78		189	78
	2			2	<u>89</u>		2	<u>9</u>	<u>7</u>

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	3	146	467	2	189	78	5	18	489
	<u>3</u>	<u>1</u>	6	2	<u>9</u>	7	5		
STEP 1	3	1	6	2	9	7	5	<u>8</u>	48
STEP 2	3	1	6	2	9	7	5	8	4

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	5	12			18			128	
	5	<u>2</u>	9	4	<u>8</u>	3	7		6
STEP 1	5	2	9	4	8	3	7	1	6

After updates of box A, row 2, box B, row 3, and row 1:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	12 2	9	4	18 8	3	7	128 1	6
row2	27 7	8	47 4	579 5	6	157 1	129 2	3	49 9
row3	3	146 1	467 6	2	189 9	78 7	5	18 8	489 4
row4	9	246	467	8	125 2	256	3	1567	57
row5	268 2	5	38	369	7	126	169	4	89
row6	678	346	1	3569	359	4	69	5678	2
row7	68	36	2	3567	358	9	4	567	1
row8	1	7	358	356	4	2568 2	26	9	35
row9	4	9	35	1	235	567	8	2567 2	357

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	1	7	358	356	4	2568 <u>2</u>	26	9	35
STEP 1	1	7	358	356	4	2568 2	26 <u>6</u>	9	35
STEP 2	1	7	358 358	356 <u>35</u>	4	2568 2	26 6	9	<u>35</u>
STEP 3	1	7	358 8	356 35	4	2568 2	26 6	9	35

-----BOX H-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	3567	358	9	3567	358	9	3567 67	358 8	9
row2	356 35	4	2568 <u>2</u>	356 <u>35</u>	4	2568 2	356 35	4	2568 2
row3	1	235	567	1	235 <u>35</u>	567	1	235 35	567 67

After updates of row 8 and box H: Note the duplicate 8's in col 5. Subpattern-1 fails, so subpattern-2 is the correct subpattern. We'll need to update the sudoku with subpattern-2 in order to solve the sudoku.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	2	9	4	18 8	3	7	128 1	6
row2	27 7	8	47 4	579 5	6	157 1	129 2	3	49 9
row3	3	14 1	467 6	2	189 9	78 7	5	18 8	489 4
row4	9	24	467	8	125 2	256	3	1567	57
row5	268 2	5	38	369	7	126	169	4	89
row6	678	36	1	3569	359	4	69	5678	2
row7	68	36	2	3567 67	358 8	9	4	567	1
row8	1	7	358 8	356 35	4	2568 2	26 6	9	35
row9	4	9	35	1	235 35	567 67	8	2567 2	357

Subpattern-2 has 2's in squares (18), (21), (42), (56), (87) and (95).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	12	9	4	18	3	7	128	6
row2	27 2	8	47	579	6	157	129	3	49
row3	3	14	467	2	189	78	5	18	489
row4	9	246 2	467	8	125	256	3	1567	57
row5	268	5	368	369	7	126	169	4	89
row6	678	36	1	3569	359	4	69	5678	2
row7	68	36	2	3567	358	9	4	567	1
row8	1	7	3568	356	4	2568	26	2	9
row9	4	9	356	1	235	567	8	2567	357

solved squares in black

	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	1	2	3	4	5	6	7	8	9
3	1	2	3	4	5	6	7	8	9
4	1	2	3	4	5	6	7	8	9
5	1	2	3	4	5	6	7	8	9
6	1	2	3	4	5	6	7	8	9
7	1	2	3	4	5	6	7	8	9
8	1	2	3	4	5	6	7	8	9
9	1	2	3	4	5	6	7	8	9

-----BOX A-----

start

	col 1	col 2	col 3
row1	5	12	9
row2	27 <u>2</u>	8	47
row3	3	14	467

step 1

	col 1	col 2	col 3
row1	5	12 <u>1</u>	9
row2	27 2	8	47
row3	3	14	467

step 2

	col 1	col 2	col 3
row1	5	12 1	9
row2	27 2	8	47
row3	3	14 <u>4</u>	467

step 3

	col 1	col 2	col 3
row1	5	1	9
row2	27 2	8	47 <u>7</u>
row3	3	14 4	467 67

step 4

	col 1	col 2	col 3
row1	5	1	9
row2	27 2	8	47 7
row3	3	14 4	467 6

		col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1	START	5	¹² <u>1</u>	9	4	18	3	7	¹²⁸ 2	6
	STEP 1	5	¹² 1	9	4	¹⁸ 8	3	7	¹²⁸ 2	6
		col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2	START	²⁷ <u>2</u>	8	⁴⁷ <u>7</u>	579	6	157	129	3	49
	STEP 1	²⁷ 2	8	⁴⁷ 7	⁵⁷⁹ <u>59</u>	6	¹⁵⁷ <u>15</u>	¹²⁹ <u>19</u>	3	49
	STEP 2	²⁷ 2	8	⁴⁷ 7	⁵⁷⁹ 59	6	¹⁵⁷ 15	¹²⁹ 19	3	⁴⁹ 4

-----BOX B-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row1	4	18 <u>8</u>	3	4	18 8	3
row2	579 59	6	157 15	579 59	6	157 15
row3	2	189	78	2	189 19	78 7

	COLUMN 1		
	start	step1	step2
row1	5	5	5
	27	27	27
row2	<u>2</u>	2	2
row3	3	3	3
row4	9	9	9
	268	268	268
row5		<u>68</u>	68
	678	678	678
row6			7
	68	<u>68</u>	68
row7			
row8	1	1	1
row9	4	4	4

-----BOX D-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row1	7	128 2	6	7	128 2	6
row2	129 19	3	49 <u>4</u>	129 19	3	49 4
row3	5	18	489	5	18	489 89

After updates of box A, row 1, row 2, box B, col 1, and box D:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	12 1	9	4	18 8	3	7	128 2	6
row2	27 2	8	47 7	579 59	6	157 15	129 19	3	49 4
row3	3	14 4	467 6	2	189 19	78 7	5	18	489
row4	9	246 2	467	8	125	256	3	1567	57
row5	268 68	5	368	369	7	126 2	169	4	89
row6	678 7	36	1	3569	359	4	69	5678	2
row7	68	36	2	3567	358	9	4	567	1
row8	1	7	3568	356	4	2568	26 2	9	35
row9	4	9	356	1	235 2	567	8	2567	357

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

COLUMN 5-----

	start	step1
row1	18 <u>8</u>	18 8
row2	6	6
row3	189 19	189 19
row4	125	125 15
row5	7	7
row6	359	359
row7	358	358 35
row8	4	4
row9	235 <u>2</u>	235 2

COLUMN 6-----

start	step1	step2
3	3	3
157 15	157 15	157 1
78 <u>7</u>	78 7	78 7
256	256 <u>56</u>	256 56
126 <u>2</u>	126 2	126 2
4	4	4
9	9	9
2568	2568 568	2568 8
567	567 <u>56</u>	567 56

-----BOX B-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	4	18 8	3	4	18 8	3	4	18 8	3
row2	579 59	6	157 <u>1</u>	579 59	6	157 1	579 5	6	157 1
row3	2	189 19	78 7	2	189 <u>9</u>	78 7	2	189 9	78 7

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	3	14 <u>4</u>	467 6	2	189 <u>9</u>	78 7	5	18	489
STEP 1	3	14 4	467 6	2	189 9	78 7	5	18	489 <u>8</u>
STEP 2	3	14 4	467 6	2	189 9	78 7	5	18 1	489 8
	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	27 2	8	47 7	579 5	6	157 <u>1</u>	129 19	3	49 4
STEP 1	27 2	8	47 7	579 5	6	157 1	129 9	3	49 4

-----BOX H-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row1	3567	358 <u>35</u>	9	3567 7	358 35	9
row2	<u>356</u>	4	2568 8	356	4	2568 8
row3	1	235 2	567 <u>56</u>	1	235 2	567 56

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	12 1	9	4	18 8	3	7	128 2	6
row2	27 2	8	47 7	579 5	6	157 1	129 9	3	49 4
row3	3	14 4	467 6	2	189 9	78 7	5	18 1	489 8
row4	9	246 2	467	8	125 15	256 56	3	1567	57
row5	268 68	5	368	369	7	126 2	169	4	89
row6	678 7	36	1	3569	359	4	69	5678	2
row7	68	36	2	3567 7	358 35	9	4	567	1
row8	1	7	3568	356	4	2568 8	26 2	9	35
row9	4	9	356	1	235 2	567 56	8	2567	357

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	68	36	2	3567 <u>7</u>	358 35	9	4	567	1
STEP 1	68	<u>36</u>	2	3567 7	358 <u>35</u>	9	4	567 <u>56</u>	1
STEP 2	68 8	36	2	3567 7	358 35	9	4	567 56	1

	COLUMN 1-----			COLUMN 5-----		
	start	step1		start	step1	step2
row1	5	5		18 8	18 8	18 8
row2	27 2	27 2		6	6	6
row3	3	3		189 <u>9</u>	189 9	189 9
row4	9	9		125 15	125 15	125 1
row5	268 68	268 6		7	7	7
row6	678 7	678 7		359	359 <u>35</u>	359 35
row7	68 <u>8</u>	68 8		358 35	358 <u>35</u>	358 35
row8	1	1		4	4	4
row9	4	4		235 2	235 2	235 2

COLUMN 3-----			COLUMN 7-----			
	start	step1		start	step1	step2
row1	9	9		7	7	7
	47	47		129	129	129
row2	<u>7</u>	7		<u>9</u>	9	9
	467	467				
row3	<u>6</u>	6		5	5	5
	467	467				
row4		4		3	3	3
	368	368		169	169	169
row5		38			16	1
				69	69	69
row6	1	1			<u>6</u>	6
row7	2	2		4	4	4
	3568	3568		26	26	26
row8		358		2	2	2
	356	356				
row9		35		8	8	8

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	678 <u>7</u>	36	1	3569	359 35	4	69 <u>6</u>	5678	2
STEP 1	678 7	36 <u>3</u>	1	3569 359	359 35	4	69 6	5678 58	2
STEP 2	678 7	36 3	1	3569 59	359 <u>5</u>	4	69 6	5678 58	2
STEP 3	678 7	36 3	1	3569 9	359 5	4	69 6	5678 8	2

After updates of row 7, col 1, col 5, col 3, col 7, and row 6:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	12 1	9	4	18 8	3	7	128 2	6
row2	27 2	8	47 7	579 5	6	157 1	129 9	3	49 4
row3	3	14 4	467 6	2	189 9	78 7	5	18 1	489 8
row4	9	246 2	467 4	8	125 1	256 56	3	1567	57
row5	268 68	5	368 38	369	7	126 2	169 1	4	89
row6	678 7	36 3	1	3569 9	359 5	4	69 6	5678 8	2
row7	68 8	36	2	3567 7	358 35	9	4	567 56	1
row8	1	7	3568 358	356	4	2568 8	26 2	9	35
row9	4	9	356 35	1	235 2	567 56	8	2567	357

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX E-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	8	125 1	256 56	8	125 1	256 <u>6</u>	8	125 1	256 6
row2	369	7	126 2	369 36	7	126 2	369 3	7	126 2
row3	3569 <u>9</u>	359 <u>5</u>	4	3569 9	359 5	4	3569 9	359 5	4

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	268 68	5	368 38	369 <u>3</u>	7	126 2	169 1	4	89
STEP 1	268 68	5	368 <u>8</u>	369 3	7	126 2	169 1	4	89
STEP 2	268 6	5	368 8	369 3	7	126 2	169 1	4	89 9

	COLUMN 2-----		COLUMN 4-----		COLUMN 5-----	
	start	step1	start	step1	start	step1
row1	12 1	12 1	 4	 4	18 8	18 8
row2	8	8	579 <u>5</u>	579 5	6	6
row3	14 4	14 4	 2	 2	189 9	189 9
row4	246 2	246 2	 8	 8	125 1	125 1
row5	 5	 5	369 <u>3</u>	369 3	 7	 7
row6	36 <u>3</u>	36 3	3569 9	3569 9	359 <u>5</u>	359 5
row7	36 6	36 6	3567 7	3567 7	358 35 3	358 35 3
row8	 7	 7	356 6	356 6	 4	 4
row9	 9	 9	 1	 1	235 2	235 2

COLUMN 6-----		
	start	step1
row1	3	3
	157	157
row2	1	1
	78	78
row3	7	7
	256	256
row4	<u>6</u>	6
	126	126
row5	2	2
row6	4	4
row7	9	9
	2568	2568
row8	8	8
	567	567
row9	56	5

After updates of box E, row 5, col 2, col 4, col 5, and col 6:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	12 1	9	4	18 8	3	7	128 2	6
row2	27 2	8	47 7	579 5	6	157 1	129 9	3	49 4
row3	3	14 4	467 6	2	189 9	78 7	5	18 1	489 8
row4	9	246 2	467 4	8	125 1	256 6	3	1567	57
row5	268 6	5	368 8	369 3	7	126 2	169 1	4	89 9
row6	678 7	36 3	1	3569 9	359 5	4	69 6	5678 8	2
row7	68 8	36 6	2	3567 7	358 3	9	4	567 56	1
row8	1	7	3568 358	356 6	4	2568 8	26 2	9	35
row9	4	9	356 35	1	235 2	567 5	8	2567	357

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	68 8	36 <u>6</u>	2	3567 7	358 3	9	4	567 56	1

STEP 1	68 8	36 6	2	3567 7	358 3	9	4	567 5	1
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-----BOX I-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	4	567 <u>5</u>	1	4	567 5	1	4	567 5	1
row2	26 <u>2</u>	9	35	26 2	9	35 <u>3</u>	26 2	9	35 3
row3	8	2567	357	8	2567 67	357 37	8	2567 67	357 <u>7</u>

step 3			
	col 1	col 2	col 3
row1	4	567 5	1
row2	26 2	9	35 3
row3	8	2567 6	357 7

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	1	7	3568 358	356 6	4	2568 <u>8</u>	26 2	9	35 <u>3</u>

STEP 1	1	7	3568 5	356 6	4	2568 8	26 2	9	35 3
--------	---	---	-----------	----------	---	-----------	---------	---	---------

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	4	9	356 35	1	235 2	567 <u>5</u>	8	2567 6	357 7

STEP 1	4	9	356 3	1	235 2	567 5	8	2567 6	357 7
--------	---	---	-----------------	---	----------	----------	---	-----------	----------

COLUMN 8

	start	step1
row1	128 2	128 2
row2	3	3
row3	18 <u>1</u>	18 1
row4	1567	1567 7
row5	4	4
row6	5678 8	5678 8
row7	567 <u>5</u>	567 5
row8	9	9
row9	2567 <u>6</u>	2567 6

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4 START	9	246 2	467 4	8	125 1	256 6	3	1567 <u>7</u>	57

STEP 1	9	246 2	467 4	8	125 1	256 6	3	1567 7	57 5
--------	---	----------	----------	---	----------	----------	---	-----------	----------------

After updates of row 7, box I, row 8, row 9, col 8 and row 4:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	12 1	9	4	18 8	3	7	128 2	6
row2	27 2	8	47 7	579 5	6	157 1	129 9	3	49 4
row3	3	14 4	467 6	2	189 9	78 7	5	18 1	489 8
row4	9	246 2	467 4	8	125 1	256 6	3	1567 7	57 5
row5	268 6	5	368 8	369 3	7	126 2	169 1	4	89 9
row6	678 7	36 3	1	3569 9	359 5	4	69 6	5678 8	2
row7	68 8	36 6	2	3567 7	358 3	9	4	567 5	1
row8	1	7	3568 5	356 6	4	2568 8	26 2	9	35 3
row9	4	9	356 3	1	235 2	567 5	8	2567 6	357 7

All rows, columns and boxes are 9-perfect, so this is the solution.

Example D.

The following sudoku has only been through trapping and annotation, plus some elementary reduction. The search for n-wings had come up with nothing for the digit 1, and the same for digit 2, but a nice simple pattern was uncovered for 2's, and the number of candidates was small, so a try was made with the 2-pattern, which is analyzed on the next page.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	248	9	468	3678	2347	78	46	1
row2	78	48	3	1	678	5	2	9	46
row3	1278	6	178	48	9	247	78	5	3
row4	134	9	15	7	1345	6	15	8	2
row5	1368	158	2	9	135	13	4	16	7
row6	146	7	156	2	145	8	159	3	69
row7	68	3	568	568	2	14	19	7	49
row8	1278	128	4	3	178	9	6	12	5
row9	9	125	1567	456	167	147	3	124	8

	1	1
1		

		2
2	2	2
	2	

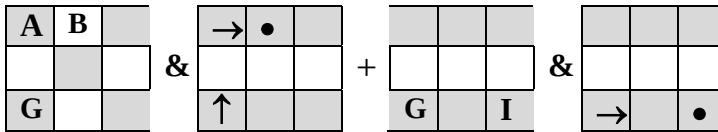
3		3
		3
3	3	3

		4
4		

5	5	5
		5

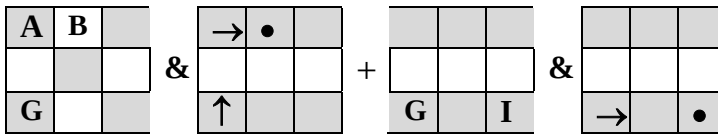
| | | | | |---|---|---| | 6 | | | | | 6 | | | | | 6 | | | | | | |---|---|---| | | | | | 7 | 7 | 7 | | | | 7 | | | | | | |--|---|---| | | | | | | 8 | 8 | | | | 8 | | | | | | |---|---|---| | 9 | 9 | 9 | | 9 | 9 | | | 9 | 9 | | |

This is the 2-pattern. We'll make box G the starting box since it has the most candidates, and choose square (81) as the beginning square of the 2-subpattern-1, choosing the order of boxes to be $G \rightarrow A \rightarrow B \rightarrow I$



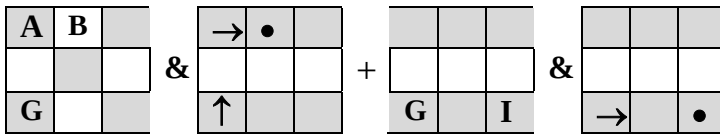
	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		2 ₁				2			
row2							X		
row3	2					2 ₁			
row4									X
row5			X						
row6				X					
row7					X				
row8	2 ₁	2						2	
row9		2						2 ₁	

Subpattern-2 is equally simple, with every choice forced.



	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		2 ₁				2 ₂			
row2							X		
row3	2 ₂					2 ₁			
row4									X
row5			X						
row6				X					
row7					X				
row8	2 ₁	2 ₂						2	
row9		2						2 ₁₂	

Subpattern-3 is equally simple, again with every choice forced.



	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		2 ₁				2 ₂₃			
row2							X		
row3	2 ₂₃					2 ₁			
row4									X
row5			X						
row6				X					
row7					X				
row8	2 ₁	2 ₂						2 ₃	
row9		2 ₃						2 ₁₂	

We'll first test subpattern-1, with 2's in squares (12), (36), (82) and (98):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	2	9						1
row2	78	48	3	1	678	5	2	9	46
row3	1278	6	178	48	9	2	78	5	3
row4	134	9	15	7	1345	6	15	8	2
row5	1368	158	2	9	135	13	4	16	7
row6	146	7	156	2	145	8	159	3	69
row7	68	3	568	568	2	14	19	7	49
row8	1278	128	4	3	178	9	6	12	5
row9	9	125	1567	456	167	147	3	124	8

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX A-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	5	<u>2</u>	9	5	2	9	5	2	9
row2	78	48	3	<u>78</u>	48	3	78	48	3
row3	1278	6	178	1278	6	<u>178</u>	1278	6	178

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	78	48	<u>4</u>	3	1	678	5	2	9
STEP 1	78	48	4	3	1	678	5	2	9
STEP 2	78	48	4	3	1	678	5	2	9

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	1278	<u>178</u>	6	178	48	9	2	78	5
STEP 1	1278	178	6	178	48	9	2	78	5

-----BOX B-----

	start			step 1		
	col 4	col 5	col 6	col 4	col 5	col 6
row1	468	3678	2347	468	3678	2347
row2	1	678	5	1	678	5
row3	48	<u>4</u>	9	48	4	9

-----BOX C-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row1	78	46	1	78	4	1
row2	2	9	<u>6</u>	2	9	6
row3	78	5	3	78	5	3

-----BOX I-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row7	19	7	49	19	7	49	<u>9</u>	7	49
row8	6	12	5	6	<u>1</u>	5	6	1	5
row9	3	124	<u>2</u>	3	2	8	3	2	8

	step 3		
	col 7	col 8	col 9
row7	9	7	4
row8	6	1	5
row9	3	2	8

After updates of box A, row 2, row 3, box B, box C and box I:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	248 2	9	468 68	3678	2347 37	78	46 4	1
row2	78	48 4	3	1	678 78	5	2	9	46 6
row3	1278 178	6	178	48 4	9	247 2	78	5	3
row4	134	9	15	7	1345	6	15	8	2
row5	1368	158	2	9	135	13	4	16	7
row6	146	7	156	2	145	8	159	3	69
row7	68	3	568	568	2	14	19 9	7	49 4
row8	1278 2	128	4	3	178	9	6	12 1	5
row9	9	125	1567	456	167	147	3	124 2	8

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	---COLUMN 9---		----COLUMN 8---	
	start	step1	start	step1
row1	1	1	46 4	46 4
row2	46 <u>6</u>	46 6	9	9
row3	3	3	5	5
row4	2	2	8	8
row5	7	7	16	16 6
row6	69	69 9	3	3
row7	49 4	49 4	7	7
row8	5	5	12 <u>1</u>	12 1
row9	8	8	124 2	124 2

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	1278 <u>2</u>	128	4	3	178	9	6	12 <u>1</u>	5
STEP 1	1278 2	128 <u>8</u>	4	3	178 78	9	6	12 1	5
STEP 2	1278 2	128 8	4	3	178 7	9	6	12 1	5

-----BOX G-----

start

	col 1	col 2	col 3
row7	68	3	568
row8	1278 <u>2</u>	128 <u>8</u>	4
row9	9	125	1567

step 1

	col 1	col 2	col 3
row7	68 <u>6</u>	3	568 <u>56</u>
row8	1278 2	128 8	4
row9	9	125 15	1567

step 2

	col 1	col 2	col 3
row7	68 <u>6</u>	3	568 <u>5</u>
row8	1278 2	128 8	4
row9	9	125 15	1567 157

step 3

	col 1	col 2	col 3
row7	68 6	3	568 5
row8	1278 2	128 8	4
row9	9	125 <u>1</u>	1567 17

step 4

	col 1	col 2	col 3
row7	68 6	3	568 5
row8	1278 2	128 8	4
row9	9	125 1	1567 7

-----BOX H-----

start

	col 4	col 5	col 6
row7	568	2	14
row8	3	178 <u>7</u>	9
row9	456	167	147

step 1

	col 4	col 5	col 6
row7	568	2	<u>14</u>
row8	3	178 7	9
row9	456	167 16	147 <u>14</u>

step 2

	col 4	col 5	col 6
row7	568	2	14
row8	3	178 7	9
row9	456 56	167 <u>6</u>	147 14

BOX H continued on next page

-----BOX H continued-----

	step 3			step 4		
	col 4	col 5	col 6	col 4	col 5	col 6
row7	568 58	2	14	568 8	2	14
row8	3	178 7	9	3	178 7	9
row9	456 <u>5</u>	167 6	147 14	456 5	167 6	147 14

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	68 6		568 5	568 8		14	19 9		49 <u>4</u>

STEP 1	68 6		568 5	568 8		14 1	19 9		49 4
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START		125 <u>1</u>	1567 7	456 5	167 6	147 14		124 2	

STEP 1		125 1	1567 7	456 5	167 6	147 4		124 2	
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After updates of col 9, col 8, row 8, box G, box H, row 7 and row 9:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	248 2	9	468 68	3678	2347 37	78	46 4	1
row2	78	48 4	3	1	678 78	5	2	9	46 6
row3	1278 178	6	178	48 4	9	247 2	78	5	3
row4	134	9	15	7	1345	6	15	8	2
row5	1368	158	2	9	135	13	4	16 6	7
row6	146	7	156	2	145	8	159	3	69 9
row7	68 6	3	568 5	568 8	2	14 1	19 9	7	49 4
row8	1278 2	128 8	4	3	178 7	9	6	12 1	5
row9	9	125 1	1567 7	456 5	167 6	147 4	3	124 2	8

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	COLUMN 2		COLUMN 3		
	start	step1	start	step1	step2
row1	248 2	248 2	9	9	9
row2	48 4	48 4	3	3	3
row3	6	6	178	178 18	178 8
row4	9	9	15	15 <u>1</u>	15 1
row5	158	158 5	2	2	2
row6	7	7	156	156 16	156 6
row7	3	3	568 <u>5</u>	568 5	568 5
row8	128 <u>8</u>	128 8	4	4	4
row9	125 <u>1</u>	125 1	1567 <u>7</u>	1567 7	1567 7

-----BOX D-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row4	134	9	<u>1</u>	134	9	1	134	9	1
row5	1368	5	2	1368	5	2	1368	5	2
row6	146	7	<u>6</u>	146	7	6	146	7	6

	step 3		
	col 1	col 2	col 3
row4	3	9	1
row5	8	5	2
row6	4	7	6

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4 START	134		15		1345		15		
	<u>3</u>	9	<u>1</u>	7		6		8	2

STEP 1	134		15		1345		15		
	3	9	1	7	45	6	<u>5</u>	8	2

STEP 2	134		15		1345		15		
	3	9	1	7	4	6	5	8	2

-----BOX F-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row4	15			15		
	<u>5</u>	8	2	5	8	2
row5		16			16	
	4	6	7	4	6	7
row6	159		69	159		69
		3	<u>9</u>	1	3	9

After updates of col 2, col 3, box D, row 4 and box F:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	248 2	9	468 68	3678	2347 37	78	46 4	1
row2	78	48 4	3	1	678 78	5	2	9	46 6
row3	1278 178	6	178 8	48 4	9	247 2	78	5	3
row4	134 3	9	15 1	7	1345 4	6	15 5	8	2
row5	1368 8	158 5	2	9	135	13	4	16 6	7
row6	146 4	7	156 6	2	145	8	159 1	3	69 9
row7	68 6	3	568 5	568 8	2	14 1	19 9	7	49 4
row8	1278 2	128 8	4	3	178 7	9	6	12 1	5
row9	9	125 1	1567 7	456 5	167 6	147 4	3	124 2	8

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	1278		178	48		247	78		
	178	6	<u>8</u>	4	9	2		5	3

STEP 1	1278		178	48		247	78		
	17	6	8	4	9	2	<u>7</u>	5	3

STEP 2	1278		178	48		247	78		
	1	6	8	4	9	2	7	5	3

	COLUMN 1		COLUMN 7		COLUMN 4	
	start	step1	start	step1	start	step1
row1	5	5	78	<u>8</u>	468 68	<u>6</u>
row2	78	<u>7</u>	2	2	1	1
row3	1278 1	1278 1	78 <u>7</u>	78 7	48 4	48 4
row4	134 3	134 3	15 5	15 5	7	7
row5	1368 <u>8</u>	1368 8	4	4	9	9
row6	146 4	146 4	159 1	159 1	2	2
row7	68 6	68 6	19 9	19 9	568 <u>8</u>	568 8
row8	1278 2	1278 2	6	6	3	3
row9	9	9	3	3	456 5	456 5

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	78 <u>7</u>	48 4	3	1	678 78	5	2	9	46 6

STEP 1	78 7	48 4	3	1	678 8	5	2	9	46 6
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	146 <u>4</u>	7	6	2	145	8	159 <u>1</u>	3	69 9

STEP 1	146 4	7	6	2	145 5	8	159 1	3	69 9
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COLUMN 6-----

	start	step1	step2
row1	2347 37	2347 37	2347 7
row2	5	5	5
row3	247 2	247 2	247 2
row4	6	6	6
row5	13	13 <u>3</u>	13 3
row6	8	8	8
row7	14 <u>1</u>	14 1	14 1
row8	9	9	9
row9	147 4	147 4	147 4

-----BOX B-----

	start			step 1		
	col 4	col 5	col 6	col 4	col 5	col 6
row1	468 <u>6</u>	3678	2347 <u>7</u>	468 6	3678 3	2347 7
row2	1	678 <u>8</u>	5	1	678 8	5
row3	48 4	9	247 2	48 4	9	247 2

-----BOX E-----

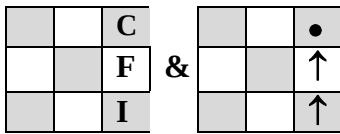
	start			step 1		
	col 4	col 5	col 6	col 4	col 5	col 6
row4	7	1345 4	6	7	1345 4	6
row5	9	135	13 <u>3</u>	9	135 1	3
row6	2	145 <u>5</u>	8	2	145 5	8

After updates of row 3, col 1, col 7, col 4, row 2, row 6, col 6, box B & box E:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	2	9	6	3	7	8	4	1
row2	7	4	3	1	8	5	2	9	6
row3	1	6	8	4	9	2	7	5	3
row4	3	9	1	7	4	6	5	8	2
row5	8	5	2	9	1	3	4	6	7
row6	4	7	6	2	5	8	1	3	9
row7	6	3	5	8	2	1	9	7	4
row8	2	8	4	3	7	9	6	1	5
row9	9	1	7	5	6	4	3	2	8

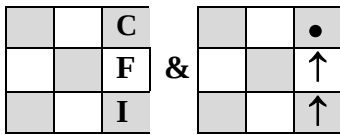
All rows, columns and boxes are 9-perfect, so subpattern-1 solves the sudoku.

Since boxes D and H are derivative boxes, the analysis can be restricted to boxes C, F, and I, with I chosen as the starting box and the order of boxes $I \rightarrow F \rightarrow C$.



Subpattern-1 is begun in square (79) of the starting box I. Because of the choice of the first square of subpattern-1, as well as the order of boxes, its extensions in boxes F and C are forced. See the next page for subpattern-2.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				X					
row2			X						
row3							9 ₁	9	9
row4						X			
row5	9								9
row6	9							9 ₁	
row7					9				9 ₁
row8		X							
row9					9		9	9	



Subpattern-2 is begun in square (98). Because of the choice of the first square of subpattern-2, as well as the order of boxes, its extensions in boxes F and C are forced. See the next page for subpattern-3.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				X					
row2			X						
row3							9 ₁₂	9	9
row4						X			
row5	9								9 ₂
row6	9							9 ₁	
row7					9				9 ₁
row8		X							
row9					9		9	9 ₂	

		C
		F
		I

&

		•
		↑
		↑

Subpattern-3 is begun in the remaining square of the starting box, namely square (97). This time there are two possible choices in box F. We arbitrarily choose square (59), labeling the box as pending. Square (38) is a forced choice in box C. This completes subpattern-3. Back at the starting box, the pending condition requires beginning a new subpattern-4. See the next page.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				X					
row2			X						
row3							9 ₁₂	9 ₃	9
row4						X			P
row5	9								9 ₂₃
row6	9							9 ₁	
row7					9				9 ₁
row8		X							
row9					9		9 ₃	9 ₂	

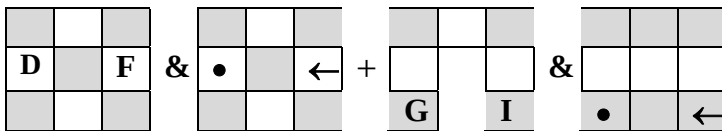
		C
		F
		I

&

		•
		↑
		↑

We begin the new subpattern-4 in the same beginning square as subpattern-3, namely (97). Going on to the next box, F, in the order of boxes, we choose the pending square (68) as the second member, and erase the pending P. Square (39) is a forced choice for the final member of subpattern-4. See the next page for the finalization of the pattern analysis.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				X					
row2			X						
row3							9 ₁₂	9 ₃	9 ₄
row4						X			P
row5	9								9 ₂₃
row6	9							9 ₁₄	
row7					9				9 ₁
row8		X							
row9					9		9 ₃₄	9 ₂	



Now that the pattern has been established for the three principal boxes, C, F and G, we establish the subscripts for the pattern members in the two derivative boxes D and G. (See the section on derivative boxes in the Subpattern Analysis Textbook, page 74.) This completes the pattern analysis.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1				X					
row2			X						
row3							9_{12}	9_3	9_4
row4						X			
row5	9_{14}								9_{23}
row6	9_{23}							9_{14}	
row7					9_{234}				9_1
row8		X							
row9					9_1		9_{34}	9_2	

We'll first test subpattern-1, with 9's in squares (37) (51), (68), (79) and (95):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	134	146	9	8	2	13	346	7
row2	367	137	9	157	4	567	8	2	56
row3	467	2	8	157	56	3	19 9	469	569
row4	1	578	257	257	235	9	6	38	4
row5	479 9	6	247	2347	1	8	27	5	39
row6	479	4578	3	6	25	457	27	89 9	1
row7	346	134	146	8	2369	46	5	7	269 9
row8	8	9	56	235	7	56	4	1	236
row9	2	3457	4567	45	3569 9	1	39	369	8

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX C-----

start

	col 7	col 8	col 9
row1	13	346	7
row2	8	2	56
row3	19	469	569
	<u>9</u>		

step 1

	col 7	col 8	col 9
row1	13	346	7
row2	8	2	<u>56</u>
row3	19	469	569
	9	46	<u>56</u>

step 2

	col 7	col 8	col 9
row1	13	346	7
row2	8	2	56
row3	19	469	569
	9	<u>4</u>	56

step 3

	col 7	col 8	col 9
row1	13	346	7
row2	8	2	56
row3	19	469	569
	9	4	56

step 4

	col 7	col 8	col 9
row1	13	346	7
row2	8	2	56
row3	19	469	569
	9	4	56

ROW 1

START

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
	5	134	146	9	8	2	13	346	7
							<u>1</u>	<u>3</u>	

STEP 1

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
	5	134	146	9	8	2	13	346	7
		<u>4</u>	46				1	3	

STEP 2

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
	5	134	146	9	8	2	13	346	7
		4	6				1	3	

-----BOX A-----

	col 1	col 2	col 3
row1	5	<u>4</u>	<u>6</u>
row2	367	137	9
row3	467	2	8

	col 1	col 2	col 3
row1	5	4	6
row2	367	137	9
row3	467	2	8

	col 1	col 2	col 3
row1	5	4	6
row2	367	137	9
row3	467	2	8

	col 1	col 2	col 3
row1	5	4	6
row2	367	137	9
row3	467	2	8

	COLUMN 1-----			COLUMN 8-----	
	start	step1	step2	step3	step4
row1	5 367	5 367	5 367	346 <u>3</u>	346 3
row2	<u>3</u> 467	3 467	3 467	2 469	2 469
row3	<u>7</u>	7	7	4 38	4 38
row4	1 479	1 479	1 479		8
row5	<u>9</u> 479	9 479	9 479	5 89	5 89
row6		<u>4</u>	4	<u>9</u>	9
row7	346	346 46	346 6	7	7
row8	8	8	8	1	1
row9	2	2	2	369	369 6

-----BOX I-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row7	5	7	269 9	5	7	269 9
row8	4	1	236	4	1	236 2
row9	39 <u>3</u>	369 <u>6</u>	8	39 3	369 6	8

After updating box C, row 1, box A, col 1, col 8, and box I:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	4	6	9	8	2	1	3	7
row2	367	137	9	157	4	567	8	2	56
row3	467	2	8	157	56	3	19	469	569
row4	1	578	257	257	235	9	6	38	4
row5	479	6	247	2347	1	8	27	5	39
row6	479	4578	3	6	25	457	27	89	1
row7	346	134	146	8	2369	46	5	7	269
row8	8	9	56	235	7	56	4	1	236
row9	2	3457	4567	45	3569	1	39	369	8

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	346 <u>6</u>	134	146	8	2369	46	5	7	269 <u>9</u>
STEP 1	346 6	134	146 14	8	2369 23	46 <u>4</u>	5	7	269 9
STEP 2	346 6	134 13	146 <u>1</u>	8	2369 23	46 4	5	7	269 9
STEP 3	346 6	134 <u>3</u>	146 1	8	2369 23	46 4	5	7	269 9
STEP 4	346 6	134 3	146 1	8	2369 2	46 4	5	7	269 9

-----BOX H-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row7	8	2369 <u>2</u>	46 <u>4</u>	8	2369 2	46 4	8	2369 2	46 4
row8	235	7	56	235 35	7	56	235 3	7	56 6
row9	45	3569 9	1	45 <u>5</u>	3569 9	1	45 5	3569 9	1

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	8	9	56	235 3	7	56 <u>6</u>	4	1	236 2
STEP 1	8	9	56 5	235 3	7	56 6	4	1	236 2

	COLUMN 5		
	start	step1	step2
row1	8	8	8
row2	4	4	4
row3	56	56	56
			6
row4	235	235	235
		35	3
row5	1	1	1
row6	25	25	25
		<u>5</u>	5
row7	2369	2369	2369
	<u>2</u>	2	2
row8	7	7	7
row9	3569	3569	3569
	9	9	9

COLUMN 4-----				
	start	step1	step2	step3
row1	9	9	9	9
	157	157	157	157
row2		<u>17</u>	17	17
	157	157	157	157
row3		<u>17</u>	17	17
	257	257	257	257
row4		27	<u>2</u>	2
	2347	2347	2347	2347
row5		247	24	4
	6	6	6	6
row6	6	6	6	6
	8	8	8	8
	235	235	235	235
row8	<u>3</u>	3	3	3
	45	45	45	45
row9	<u>5</u>	5	5	5

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	467 <u>7</u>	2	8	157 17	56 <u>6</u>	3	19 9	469 4	569 56

STEP 1	467 7	2	8	157 1	56 6	3	19 9	469 4	569 5
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	479 <u>4</u>	4578	3	6	25 <u>5</u>	457	27	89 9	1

STEP 1	479 4	4578 78	3	6	25 5	457 <u>7</u>	27	89 9	1
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STEP 2	479 4	4578 8	3	6	25 5	457 7	27 2	89 9	1
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After updating row 7, box H, row 8, col 5, col 4, row 3, and row 6:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	4	6	9	8	2	1	3	7
row2	367 3	137 1	146 9	157 17	4	567	8	2	56
row3	467 7	2	8	157 1	56 6	3	19 9	469 4	569 5
row4	1	578	257	257 2	235 3	9	6	38 8	4
row5	479 9	6	247	2347 4	1	8	27	5	39
row6	479 4	4578 8	3	6	25 5	457 7	27 2	89 9	1
row7	346 6	134 3	146 1	8	2369 2	46 4	5	7	269 9
row8	8	9	56 5	235 3	7	56 6	4	1	236 2
row9	2	3457	4567	45 5	3569 9	1	39 3	369 6	8

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	COLUMN 2-----		COLUMN 3-----		COLUMN 4-----	
	start	step1	step2	step3	step4	step5
row1	134 <u>4</u>	134 4	146 <u>6</u>	146 6	9	9
row2	137 1	137 1	9	9	157 17	157 7
row3	2	2	8	8	157 <u>1</u>	157 1
row4	578	578 57	257	257 27	257 2	257 2
row5	6	6	247	247	2347 4	2347 4
row6	4578 <u>8</u>	4578 8	3	3	6	6
row7	134 <u>3</u>	134 3	146 1	146 1	8	8
row8	9	9	56 <u>5</u>	56 5	235 3	235 3
row9	3457	3457 57	4567	4567 47	45 5	45 5

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	2	3457 57	4567 47	45 <u>5</u>	3569 9	1	39 3	369 6	8

STEP 1	2	3457 <u>7</u>	4567 47	45 5	3569 9	1	39 3	369 6	8
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STEP 2	2	3457 7	4567 <u>4</u>	45 5	3569 9	1	39 3	369 6	8
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	479 <u>9</u>	6	247	2347 <u>4</u>	1	8	27	5	39

STEP 1	479 9	6	247 27	2347 4	1	8	27	5	39 3
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4 START	1	578 57	257 27	257 <u>2</u>	235 3	9	6	38 8	4

STEP 1	1	578 57	257 <u>7</u>	257 2	235 3	9	6	38 8	4
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STEP 2	1	578 5	257 7	257 2	235 3	9	6	38 8	4
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	COLUMN 3-----		COLUMN 6-----		COLUMN 7-----	
	start	step1	start	step1	start	step1
row1	146 6	146 6	2	2	13 1	13 1
row2	9	9	567	567 5	8	8
row3	8	8	3	3	19 9	19 9
row4	257 <u>7</u>	257 7	9	9	6	6
row5	247 27	247 2	8	8	27	27 7
row6	3	3	457 <u>7</u>	457 7	27 <u>2</u>	27 2
row7	146 1	146 1	46 4	46 4	5	5
row8	56 5	56 5	56 <u>6</u>	56 6	4	4
row9	4567 4	4567 4	1	1	39 3	39 3

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	367 3	137 1	9	157 7	4	567 <u>5</u>	8	2	56

STEP 1	367 3	137 1	9	157 7	4	567 5	8	2	56 6
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After updates of col 2/ col 3, col 4, row 9, row 5, row , col 3, col 6, col 7, and row 2: All rows, columns and boxes are 9-perfect, so subpattern-1 solves the sudoku.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	4	6	9	8	2	1	3	7
row2	3	1	9	7	4	5	8	2	6
row3	7	2	8	1	6	3	9	4	5
row4	1	5	7	2	3	9	6	8	4
row5	9	6	2	4	1	8	7	5	3
row6	4	8	3	6	5	7	2	9	1
row7	6	3	1	8	2	4	5	7	9
row8	8	9	5	3	7	6	4	1	2
row9	2	7	4	5	9	1	3	6	8

Example F.

This sudoku has been through trapping, annotation, and a fruitless search for n-wings.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7	8	3	29	12	129	6	5	4
row2	14	14	5	6	8	37	27	2379	29
row3	6	2	9	5	37	4	8	37	1
row4	8	3	2	79	6	159	4	179	59
row5	19	5	17	3	4	129	127	8	6
row6	149	147	6	279	125	8	3	127	259
row7	3	9	47	1	57	6	25	24	8
row8	2	167	147	8	9	57	15	146	3
row9	5	16	8	4	23	23	9	16	7

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The 5-pattern looks promising. We'll analyze it on the next page.

We'll choose the order of boxes to be $F \rightarrow E \rightarrow H \rightarrow I$. We choose square (49) as the beginning square of box F. Its extension in E is (65), and in H it is (86), while in I it is (77). Back at the starting box, one square (69) is left with which to begin subpattern-2, namely (69). All of its extensions are forced, with (69) in F, (75) in H, and (87) in I.

	E	F
	H	I

	↓	←
	→	•

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1								X	
row2			X						
row3				X					
row4						5 ₂			5 ₁
row5		X							
row6					5 ₁				5 ₂
row7					5 ₂		5 ₁		
row8						5 ₁	5 ₂		
row9	X								

We'll begin the testing with subpattern-1, with 5's in squares (49), (65), (77) and (86):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7	8	3	29	12	129	6	5	4
row2	14	14	5	6	8	37	27	2379	29
row3	6	2	9	5	37	4	8	37	1
row4	8	3	2	79	6	159	4	179	59
row5	19	5	17	3	4	129	127	8	6
row6	149	147	6	279	125	8	3	127	259
row7	3	9	47	1	57	6	25	24	8
row8	2	167	147	8	9	57	15	146	3
row9	5	16	8	4	23	23	9	16	7

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	3	9	47	1	57	6	25 <u>5</u>	24	8
STEP 1	3	9	47	1	<u>7</u>	6	25 5	24	8
STEP 2	3	9	47 <u>4</u>	1	7	6	25 5	24	8
STEP 3	3	9	47 4	1	7	6	25 5	24 2	8

-----BOX I-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row7	25 <u>5</u>	24 2	col 9 8	25 5	24 2	col 9 8	25 5	24 2	col 9 8
row8	15	146	3	15	146	3	15	146 46	3
row9	9	16	7	9	16	7	9	16 <u>6</u>	7
	step 3								
	col 7	col 8	col 9						
row7	25 5	24 2	col 9 8						
row8	15 1	146 4	3						
row9	9	16 6	7						

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	2	167	147	8	9	57	15 <u>1</u>	146 <u>4</u>	3
STEP 1	2	167 67	147 <u>7</u>	8	9	57	15 1	146 4	3
STEP 2	2	167 6	147 7	8	9	57	15 1	146 4	3

	COLUMN 5 -----				COLUMN 3 -----	
	start	step1	step2	step3	start	step1
row1	12	12	12	12 1	3	3
row2	8	8	8	8	5	5
row3	37	37 <u>3</u>	37 3	37 3	9	9
row4	6	6	6	6	2	2
row5	4	4	4	4	17	17 1
row6	125 5	125 5	125 5	125 5	6	6
row7	57 <u>7</u>	57 7	57 7	57 7	47 4	47 4
row8	9	9	9	9	147 <u>7</u>	147 7
row9	23	23	23 <u>2</u>	23 2	8	8

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	5	16	8	4	23 <u>2</u>	23	9	16 <u>6</u>	7
STEP 1	5	16 1	8	4	23 2	23 3	9	16 6	7

After updates of row 7, box I, row 8, col 5, col 3 and row 9:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7	8	3	29	12	129	6	5	4
row2	14	14	5	6	8	37	27	2379	29
row3	6	2	9	5	37	4	8	37	1
row4	8	3	2	79	6	159	4	179	59
row5	19	5	1	3	4	129	127	8	6
row6	149	147	6	279	125	8	3	127	259
row7	3	9	47	1	57	6	25	24	8
row8	2	167	147	8	9	57	15	146	3
row9	5	16	8	4	23	23	9	16	7

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	19	5	<u>1</u>	3	4	129	127	8	6
STEP 1	19 <u>9</u>	5	1	3	4	129 29	127 27	8	6
STEP 2	19 9	5	1	3	4	129 <u>2</u>	127 27	8	6
STEP 3	19 9	5	1	3	4	129 2	127 7	8	6

-----BOX D-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row4	8	3	2	8	3	2	8	3	2
row5	19 <u>9</u>	5	<u>1</u>	19 9	5	1	19 9	5	1
row6	149 4	147	6	149 <u>4</u>	147 47	6	149 4	147 7	6

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	149	147	6	279	125	8	3	127	259
	4	<u>7</u>	6		<u>5</u>				
STEP 1	149	147	6	279	125	8	3	127	259
	4	7	6	<u>29</u>	5			12	<u>29</u>
STEP 2	149	147	6	279	125	8	3	127	259
	4	7	6	29	5			1	29

-----BOX F-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row4	4	179	<u>5</u>	4	<u>9</u>	5	4	9	5
row5	127 <u>7</u>	8	6	127 7	8	6	127 7	8	6
row6	3	127 <u>1</u>	259	3	127 1	259 <u>29</u>	3	127 1	259 <u>2</u>

-----BOX E-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row4	79	6	159	79	6	<u>19</u>	79	6	<u>1</u>
row5	3	4	129 <u>2</u>	3	4	2	3	4	2
row6	279 29	125 <u>5</u>	8	279 <u>9</u>	125 5	8	279 9	125 5	8

After updating row 5, box D, row 6, box F and box E:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7	8	3	29	12	129	6	5	4
row2	14	14	5	6	8	37	27	2379	29
row3	6	2	9	5	37	4	8	37	1
row4	8	3	2	79	6	159	4	179	59
row5	19	5	17	3	4	129	127	8	6
row6	149	147	6	279	125	8	3	127	259
row7	3	9	47	1	57	6	25	24	8
row8	2	167	147	8	9	57	15	146	3
row9	5	16	8	4	23	23	9	16	7

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	COLUMN 1-----		COLUMN 2-----		COLUMN 4-----	
	start	step1	start	step1	start	step1
row1	7	7	8	8	29	2
	14	14	14	14		
row2		1		4	6	6
row3	6	6	2	2	5	5
row4	8	8	3	3	79	7
	19	19				
row5	9	9	5	5	3	3
	149	149	147	147	279	279
row6	<u>4</u>	4	7	7	<u>9</u>	9
row7	3	3	9	9	1	1
			167	167		
row8	2	2	6	6	8	8
			16	16		
row9	5	5	<u>1</u>	1	4	4

	COLUMN 6-----		COLUMN 7-----		COLUMN 9	
	start	step1	step2	step3	step4	step5
row1	129	129 9	6	6	4	4
row2	37	37 7	27	27 2	29	29 9
row3	4	4	8	8	1	1
row4	159 <u>1</u>	159 1	4	4	59 5	59 5
row5	129 <u>2</u>	129 2	127 <u>7</u>	127 7	6	6
row6	8	8	3	3	259 <u>2</u>	259 2
row7	6	6	25 5	25 5	8	8
row8	57 5	57 5	15 1	15 1	3	3
row9	23 <u>3</u>	23 3	9	9	7	7

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	14 1	14 4	5	6	8	37 <u>7</u>	27 <u>2</u>	2379	29 <u>9</u>

STEP 1	14 1	14 4	5	6	8	37 7	27 2	2379 3	29 9
--------	---------	---------	---	---	---	---------	---------	------------------	---------

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	6	2	9	5	37 <u>3</u>	4	8	37	1

STEP 1	6	2	9	5	37 3	4	8	37 7	1
--------	---	---	---	---	---------	---	---	---------	---

After updating col 1, col 2, col 3, col 6, col 7, col 9, row 2 and row 3:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	7	8	3	29 2	12 1	129 9	6	5	4
row2	14 1	14 4	5	6	8	37 7	27 2	2379 3	29 9
row3	6	2	9	5	37 3	4	8	37 7	1
row4	8	3	2	79 7	6	159 1	4	179 9	59 5
row5	19 9	5	17 1	3	4	129 2	127 7	8	6
row6	149 4	147 7	6	279 9	125 5	8	3	127 1	259 2
row7	3	9	47 4	1	57 7	6	25 5	24 2	8
row8	2	167 6	147 7	8	9	57 5	15 1	146 4	3
row9	5	16 1	8	4	23 2	23 3	9	16 6	7

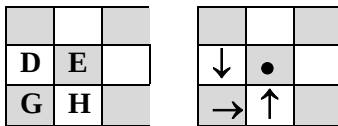
All rows, columns and boxes are 9-perfect, so subpattern-1 solves the sudoku.

Example G.

This sudoku went through trapping and annotation, plus the discovery of both a 2-wing and a 3-wing, but when the analysis of the 4's offered a reasonably simple 4-pattern, it was decided to try to finish it off with the 4-subpattern.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	67	1	8	359	57	3679	4	2367	267
row2	67	2	9	8	37	4	36	5	1
row3	3	4	5	1	2	67	68	678	9
row4	9	7	1	2	6	5	38	4	38
row5	25	8	6	349	34	39	1	27	257
row6	45	3	24	7	1	8	256	9	56
row7	1	6	27	35	9	37	258	38	4
row8	48	5	47	6	3478	2	9	1	38
row9	248	9	3	45	458	1	7	268	2568

1	1	1	2	2		3			4	4	4	5		5
1	1	1		2		3					4		5	
1	1	1		2		3					4	5		
									8	8		9		9
6	6			7	7				8	8		9		9
6	6					7						9	9	9



This is the 4-pattern. We'll make D the starting box with the order of boxes

D→G→H→E. We'll choose (61) as the beginning square of subpattern-1. In the second box G, there is only one square not in the neighborhood of subpattern-1 – e.g. not in the same column as square (61), and that is (83). When we get to box H, there are two choices. We choose (94) as the third member of subpattern-1, mark box H as pending, and append the subscript 0 to the pending square (95). In the last box, E, there is only one square not in the neighborhood of subpattern-1, namely (55). This completes subpattern-1.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1							X		
row2						X			
row3		X							
row4									
row5				4	4 ₁				
row6	4 ₁		4						
row7						P ₁			
row8	4		4 ₁		4				
row9	4			4 ₁	4 ₀				

D	E			
G	H			

↓	•			
→	↑			

Back at the starting box D, we note that there is a pending condition, so we begin subpattern-2 in the same square as subpattern-1. We do the same in the next box, G. The third box, H, is the pending box, so we assign the pending square to subpattern-2, erase the pending P, In the final box, there is only one square, (54), not in the neighborhood of subpattern-2, so it becomes the final member of subpattern-2.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1							X		
row2						X			
row3		X							
row4									
row5				4 ₂	4 ₁				
row6	4 ₁₂		4						
row7						P₁			
row8	4		4 ₁₂		4				
row9	4			4 ₁	4 ₂				

D	E			
G	H			

↓	•			
→	↑			

Back at the starting box D, there is no pending situation, so we begin subpattern-3 in the only remaining square, (63). When we get to box G, we have two choices, so we choose one of them, (81), mark the box as pending, and append a zero subscript to the pending square. When we get to box H, we again have a pending situation, so we choose square (94) as the next member of subpattern-3, append a zero subscript to the pending square, and mark the box as pending. When we get to the final box, there is only one choice, (55) as the final member of subpattern-3.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1							X		
row2						X			
row3		X							
row4									
row5				4 ₂	4 ₁₃				
row6	4 ₁₂		4 ₃						
row7	P ₂								
row8	4 ₃		4 ₁₂		4	P ₃			
row9	4 ₀			4 ₁₃	4 ₂₀				

D	E			
G	H			

↓	•			
→	↑			

Back at the starting box D, there is a pending condition, so we begin subpattern-4 in the same square as subpattern-3, namely (63). Since box G is not the highest pending box, we again assign subpattern-4 to the same square as subpattern-3. At square H, the highest pending box, we assign the pending square (95), and erase the pending P3. In box E, there is only one square not in the neighborhood of subpattern-4, namely (54), which becomes the final member of subpattern-4.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1							X		
row2						X			
row3		X							
row4									
row5				4_{24}	4_{13}				
row6	4_{12}		4_{34}						
row7	P_2								
row8	4_{34}		4_{12}		4	P_3			
row9	4_0			4_{13}	4_{24}				

D	E				
G	H				

↓	•				
→	↑				

Back at the starting box D, there is a pending condition, so we begin a new subpattern-5 in the same beginning square as subpattern-4, namely (63). The next box is the highest pending box (also the only), so the pending square, (91) becomes the second member of subpattern-5. In box H, there is only one square not yet in the neighborhood of subpattern-5, namely (85), which then becomes the third member of subpattern-5. Finally, in box E, there is only one choice for subpattern-5, namely square (54). This completes the analysis of the subpatterns for the digit 4. Next we will test them out to find out which is the correct one.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1							X		
row2						X			
row3		X							
row4									
row5				4 ₂₄₅	4 ₁₃				
row6	4 ₁₂		4 ₃₄₅						
row7	P₂								
row8	4 ₃₄		4 ₁₂		4 ₅				
row9	4 ₅			4 ₁₃	4 ₂₄				

We'll first test subpattern-1, with 4's in squares (55), (61), (83), and (94):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	67	1	8	359	57	3679	4	2367	267
row2	67	2	9	8	37	4	36	5	1
row3	3	4	5	1	2	67	68	678	9
row4	9	7	1	2	6	5	38	4	38
row5	25	8	6	349	34	39	1	27	257
row6	45	3	24	7	1	8	256	9	56
row7	1	6	27	35	9	37	258	38	4
row8	48	5	47	6	3478	2	9	1	38
row9	248	9	3	45	458	1	7	268	2568

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX D-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row4	9	7	1	9	7	1	9	7	1
	25			25			25		
row5		8	6		8	6	5	8	6
	45		24	45		24	45		24
row6	<u>4</u>	3		4	3	<u>2</u>	4	3	2

-----BOX G-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row7	1	6	27	1	6	27	1	6	27
	48		47	48		47	48		47
row8		5	<u>4</u>	<u>8</u>	5	4	8	5	4
	248			248			248		
row9		9	3	28	9	3	<u>2</u>	9	3

	step 3		
	col 1	col 2	col 3
row7	1	6	7
	48		47
row8	8	5	4
	248		
row9	2	9	3

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	1	6	<u>7</u>	35	9	37	258	38	4
STEP 1	1	6	7	35	9	<u>3</u>	258	38	4
STEP 2	1	6	7	<u>5</u>	9	3	258	<u>8</u>	4
STEP 3	1	6	7	5	9	3	258	8	4

-----BOX I-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row7	258 <u>2</u>	38 <u>8</u>		258 2	38 8		258 2	38 8	
row8	9	1	38	9	1	38	9	1	38
row9	7	268	2568	7	268	2568 56	7	268	2568 5

-----BOX H-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row7	35		37	35		37	35		37
	<u>5</u>	9	<u>3</u>	5	9	3	5	9	3
row8		3478			3478			3478	
	6		2	6	78	2	6	7	2
row9	45	458		45	458		45	458	
	4		1	4	8	1	4	8	1

	COLUMN 5-----		COLUMN 9-----		
	start	step1	start	step1	step2
row1	57	57 5	267	267	267 27
row2	37	37 3	1	1	1
row3	2	2	9	9	9
row4	6	6	38	38 8	38 8
row5	34 4	34 4	257	257 27	257 27
row6	1	1	56	56 <u>6</u>	56 6
row7	9	9	4	4	4
row8	3478 <u>7</u>	3478 7	38 <u>3</u>	38 3	38 3
row9	458 8	458 8	2568 <u>5</u>	2568 5	2568 5

After updating box D, box G, row 7, box I, box H, col 5, and col 9:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	67 1	8	359	57 5	3679	4	2367	267 27	
row2	67 2	9	8	37 3	4	36	5	1	
row3	3	4	5	1	2	67	68	678 9	
row4	9	7	1	2	6	5	38	4 38 8	
row5	25 5	8	6	349	34 4	39	1	27 257 27	
row6	45 4	3	24 2	7	1	8	256	9 56 6	
row7	1	6	27 7	35 5	9	37 3	258 2	38 8	4
row8	48 8	5	47 4	6	3478 7	2	9	1 38 3	
row9	248 2	9	3	45 4	458 8	1	7	268 6	2568 5

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	45 4		24 <u>2</u>	7	1	8	256	9	56 <u>6</u>

STEP 1	45 4	3	24 2	7	1	8	256 5	9	56 6
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4 START	9	7	1	2	6	5	38	4	38 <u>8</u>

STEP 1	9	7	1	2	6	5	38 3	4	38 8
--------	---	---	---	---	---	---	---------	---	---------

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	67	2	9	8	37 <u>3</u>	4	36	5	1

STEP 1	67	2	9	8	37 3	4	36 <u>6</u>	5	1
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STEP 2	67 7	2	9	8	37 3	4	36 6	5	1
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-----BOX C-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row1	4	2367	267 27	4	2367 237	267 27	4	2367 237	267 27
row2	36 <u>6</u>	5	1	36 6	5	1	36 6	5	1
row3	68	678	9	68 <u>8</u>	678 78	9	68 8	678 <u>7</u>	9

BOX C continued on next page

-----BOX C continued-----

step 3

	col 7	col 8	col 9
row1	4	2367 23	267 <u>2</u>
row2	36 6	5	1
row3	68 8	678 7	9

step 4

	col 7	col 8	col 9
row1	4	2367 3	267 2
row2	36 6	5	1
row3	68 8	678 7	9

COLUMN 1-----

	start	step1
row1	67	67 6
row2	67 <u>7</u>	67 7
row3	3	3
row4	9	9
row5	25 5	25 5
row6	45 4	45 4
row7	1	1
row8	48 8	48 8
row9	248 2	248 2

COLUMN 6-----

	start	step1	step2
row1	3679	3679 679	3679 67
row2	4	4	4
row3	67	67	67
row4	5	5	5
row5	39	39 <u>9</u>	39 9
row6	8	8	8
row7	37 <u>3</u>	37 3	37 3
row8	2	2	2
row9	1	1	1

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	67 <u>6</u>	1	8	359	57 <u>5</u>	3679 67	4	2367 <u>3</u>	267 2

STEP 1	67 6	1	8	359 9	57 5	3679 7	4	2367 3	267 2
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	COLUMN 4-----		COLUMN 6-----		COLUMN 8-----	
	start	step1	start	step1	start	step1
row1	359 <u>9</u>	359 9	3679 <u>7</u>	3679 7	2367 3	2367 3
row2	8	8	4	4	5	5
row3	1	1	67	67 <u>6</u>	678 <u>7</u>	678 7
row4	2	2	5	5	4	4
row5	349	349 <u>3</u>	39	39 9	27	27 2
row6	7	7	8	8	9	9
row7	35 5	35 5	37 3	37 3	38 8	38 8
row8	6	6	2	2	1	1
row9	45 <u>4</u>	45 4	1	1	268 6	268 6

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	25 5	8	6	349 3	34 4	39 9	1	27 <u>2</u>	257 27

STEP 1	25 5	8	6	349 3	34 4	39 9	1	27 2	257 7
--------	---------	---	---	----------	---------	---------	---	---------	----------

A check of all the rows, columns and boxes shows them all to be 9-perfect. Therefore subpattern-1 solves the sudoku.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	⁶⁷ 6	1	8	³⁵⁹ 9	⁵⁷ 5	³⁶⁷⁹ 7	4	²³⁶⁷ 3	²⁶⁷ 2
row2	⁶⁷ 7	2	9	8	³⁷ 3	4	³⁶ 6	5	1
row3	3	4	5	1	2	⁶⁷ 6	⁶⁸ 8	⁶⁷⁸ 7	9
row4	9	7	1	2	6	5	³⁸ 3	4	³⁸ 8
row5	²⁵ 5	8	6	³⁴⁹ 3	³⁴ 4	³⁹ 9	1	²⁷ 2	²⁵⁷ 7
row6	⁴⁵ 4	3	²⁴ 2	7	1	8	²⁵⁶ 5	9	⁵⁶ 6
row7	1	6	²⁷ 7	³⁵ 5	9	³⁷ 3	²⁵⁸ 2	³⁸ 8	4
row8	⁴⁸ 8	5	⁴⁷ 4	6	³⁴⁷⁸ 7	2	9	1	³⁸ 3
row9	²⁴⁸ 2	9	3	⁴⁵ 4	⁴⁵⁸ 8	1	7	²⁶⁸ 6	²⁵⁶⁸ 5

Example H.

This sudoku went through trapping, annotation, and simple updates related to annotation. The 1-pattern was examined for n-wings, but instead of checking all digits for n-wings, and reducing the sudoku as much as possible before resorting to a subpattern analysis, the 1-pattern looked simple enough that an attempt was made to try it out immediately.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	136	36	2	167	17	9	48	48	5
row2	146	8	46	5	3	16	9	7	2
row3	9	7	5	4	2	8	6	3	1
row4	2	34569	3469	8	459	347	1	456	367
row5	3458	1	3489	39	6	347	4578	2	378
row6	34568	3456	7	13	145	2	458	4568	9
row7	368	369	1	3679	79	5	2	689	4
row8	3456	2	3469	369	8	346	57	1	67
row9	7	4569	4689	2	149	146	3	5689	68

			2	2	2		3	3		4		5	5	5
1		1	2	2	2									
1		1	2	2	2			3			4		5	

		6	7		7	8	8		9	9	9
	6		7				8				9
			7				8				

A	B	C
D	E	F
G	H	I

This is the 1-pattern. We'll choose B for the starting box, with the initial order of boxes **B→A→E→H**. Subpattern-1 is too simple to explain.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	1			1 ₁	1				
row2	1 ₁					1			
row3									
row4									
row5									
row6				1	1 ₁				
row7									
row8									
row9					1	1 ₁			

A	B	C
D	E	F
G	H	I

Using the same box order, subpattern-2 quickly follows:.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	1			1 ₁	1 ₂				
row2	1 ₁₂					1			
row3									
row4									
row5									
row6				1 ₂	1 ₁				
row7									
row8									
row9					1	1 ₁₂			

A	B	C
D	E	F
G	H	I

Subpattern-3 follows just as quickly: Since no pending situations exist, we are allowed to change the box order to $B \rightarrow A \rightarrow H \rightarrow E$, which leads quickly to the following solution:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	1 ₃			1 ₁	1 ₂				
row2	1 ₁₂					1 ₃			
row3									
row4									
row5									
row6				1 ₂₃	1 ₁				
row7									
row8									
row9					1 ₃	1 ₁₂			

We'll first test subpattern-1, with 1's in squares (14), (21), (65), and (96):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	136	36	2	167 1	17	9	48	48	5
row2	146 1	8	46	5	3	16	9	7	2
row3	9	7	5	4	2	8	6	3	1
row4	2	34569	3469	8	459	347	1	456	367
row5	3458	1	3489	39	6	347	4578	2	378
row6	34568	3456	7	13	145 1	2	458	4568	9
row7	368	369	1	3679	79	5	2	689	4
row8	3456	2	3469	369	8	346	57	1	67
row9	7	4569	4689	2	149	146 1	3	5689	68

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX A-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	136	36		136	<u>36</u>		136	36	
row2	146		46	146		46	146		46
	<u>1</u>	8		1	8		1	8	<u>4</u>
row3	9	7	5	9	7	5	9	7	5

-----BOX B-----

	start			step 1		
	col 4	col 5	col 6	col 4	col 5	col 6
row1	167	17		167	17	
	<u>1</u>		9	1	<u>7</u>	9
row2			16			16
	5	3		5	3	<u>6</u>
row3	4	2	8	4	2	8

-----BOX E-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row4		459	347		459	347		459	<u>47</u>
row5	39		347	39		347	39		347
		6			6		<u>9</u>	6	<u>47</u>
row6	13	145		13	145		13	145	
		<u>1</u>	2	<u>3</u>	1	2	3	1	2
	step 3								
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row4	8	5	47	8	5	47	8	5	47
row5	9	6	47	9	6	47	9	6	47
row6	3	1	2	3	1	2	3	1	2

	COLUMN 4-----			COLUMN 5-----		
	start	step1	step2	start	step1	step2
row1	167 1	167 1	167 1	17 <u>7</u>	17 7	17 7
row2	5	5	5	3	3	3
row3	4	4	4	2	2	2
row4	8	8	8	459 5	459 5	459 5
row5	39 <u>9</u>	39 <u>9</u>	39 <u>9</u>	6	6	6
row6	13 <u>3</u>	13 3	13 3	145 <u>1</u>	145 1	145 1
row7	3679	3679 67	3679 7	79	79 <u>9</u>	79 9
row8	369	369 <u>6</u>	369 6	8	8	8
row9	2	2	2	149	149 49	149 4

	-----BOX H-----		
	start		
	col 4	col 5	col 6
row7	3679 7	79 9	col 6 5
row8	369 <u>6</u>	8	346
row9	2	149 <u>4</u>	146 1

	step 1		
	col 4	col 5	col 6
row7	3679 7	79 9	col 6 5
row8	369 6	8	346 3
row9	2	149 4	146 1

After updating box A, box B, box E, col 4 , col 5, and box H

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	136 36	36	2	167 1	17 7	9	48	48	5
row2	146 1	8	46 4	5	3	16 6	9	7	2
row3	9	7	5	4	2	8	6	3	1
row4	2	34569	3469	8	459 5	347 47	1	456	367
row5	3458	1	3489	39 9	6	347 47	4578	2	378
row6	34568	3456	7	13 3	145 1	2	458	4568	9
row7	368	369	1	3679 7	79 9	5	2	689	4
row8	3456	2	3469	369 6	8	346 3	57	1	67
row9	7	4569	4689	2	149 4	146 1	3	5689	68

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	3456	2	3469	369 <u>6</u>	8	346 <u>3</u>	57	1	67
STEP 1	3456 45	2	3469 49	369 6	8	346 3	57	1	67 <u>7</u>
STEP 2	3456 45	2	3469 49	369 6	8	346 3	57 <u>5</u>	1	67 7
STEP 3	3456 <u>4</u>	2	3469 49	369 6	8	346 3	57 5	1	67 7
STEP 4	3456 4	2	3469 9	369 6	8	346 3	57 5	1	67 7

	start	step1	step2
row1	48	<u>48</u>	48
row2	9	9	9
row3	6	6	6
row4	1	1	1
row5	4578	4578 478	4578 7
row6	458	458 <u>48</u>	458 48
row7	2	2	2
row8	57 <u>5</u>	57 5	57 5
row9	3	3	3

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	3458	1	3489	39 <u>9</u>	6	347 47	4578 <u>7</u>	2	378
STEP 1	3458	1	3489 348	39 9	6	347 <u>4</u>	4578 7	2	378 38
STEP 2	3458 358	1	3489 <u>38</u>	39 9	6	347 4	4578 7	2	378 <u>38</u>
STEP 3	3458 5	1	3489 38	39 9	6	347 4	4578 7	2	378 38

-----BOX G-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row7	368	369	1	<u>368</u>	369	1	368	369 36	1
row8	3456 <u>4</u>	2	3469 <u>9</u>	3456 4	2	3469 9	3456 4	2	3469 9
row9	7	4569	4689	7	4569 56	4689 <u>68</u>	7	4569 5	4689 68

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	7	4569 <u>5</u>	4689 <u>68</u>	2	149 4	146 1	3	5689	<u>68</u>
STEP 1	7	4569 5	4689 68	2	149 4	146 1	3	5689 9	68

	COLUMN 2-----		
	start	step1	step2
row1	<u>36</u>	36	36
row2	8	8	8
row3	7	7	7
row4	34569	34569 49	34569 9
row5	1	1	1
row6	3456	3456 4	3456 4
row7	369 <u>36</u>	369 36	369 36
row8	2	2	2
row9	4569 <u>5</u>	4569 5	4569 5

After updates of row 8, col 7, row 5, box G, row 9, and col 2:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	136 36	36	2	167 1	17 7	9	48	48	5
row2	146 1	8	46 4	5	3	16 6	9	7	2
row3	9	7	5	4	2	8	6	3	1
row4	2	34569 9	3469	8	459 5	347 47	1	456	367
row5	3458 5	1	3489 38	39 9	6	347 4	4578 7	2	378 38
row6	34568	3456 4	7	13 3	145 1	2	458 48	4568	9
row7	368	369 36	1	3679 7	79 9	5	2	689	4
row8	3456 4	2	3469 9	369 6	8	346 3	57 5	1	67 7
row9	7	4569 5	4689 68	2	149 4	146 1	3	5689 9	68

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX D-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row4	2	<u>9</u>		2	9	36
row5	3458		3489	3458		3489
	<u>5</u>	1	38	5	1	38
row6	34568	3456		34568	3456	
		<u>4</u>	7	368	4	7

-----BOX I-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row7	2	689	4	2	68	4
row8	57		67	57		67
	5	1	7	5	1	7
row9	3	5689	68	3	5689	68
		<u>9</u>			9	

-----BOX F-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row4	1	456	367	1	456	36
row5	4578		378	4578		378
	<u>7</u>	2	38	7	2	38
row6	458	4568		458	4568	
	48		9	48		9

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4 START	2	³⁴⁵⁶⁹ 9	³⁴⁶⁹ <u>36</u>	8	⁴⁵⁹ <u>5</u>	³⁴⁷ 47	1	456	³⁶⁷ <u>36</u>
STEP 1	2	³⁴⁵⁶⁹ 9	³⁴⁶⁹ 36	8	⁴⁵⁹ 5	³⁴⁷ 47	1	456 <u>4</u>	³⁶⁷ 36
STEP 2	2	³⁴⁵⁶⁹ 9	³⁴⁶⁹ 36	8	⁴⁵⁹ 5	³⁴⁷ 7	1	456 4	³⁶⁷ 36

	COLUMN 8 -----			
	start	step1	step2	step3
row1	48	48 <u>8</u>	48 8	48 8
row2	7	7	7	7
row3	3	3	3	3
row4	⁴⁵⁶ <u>4</u>	⁴⁵⁶ 4	⁴⁵⁶ 4	⁴⁵⁶ 4
row5	2	2	2	2
row6	⁴⁵⁶⁸	⁴⁵⁶⁸ 568	⁴⁵⁶⁸ 56	⁴⁵⁶⁸ 5
row7	⁶⁸⁹ 68	⁶⁸⁹ 68	⁶⁸⁹ <u>6</u>	⁶⁸⁹ 6
row8	1	1	1	1
row9	⁵⁶⁸⁹ 9	⁵⁶⁸⁹ 9	⁵⁶⁸⁹ 9	⁵⁶⁸⁹ 9

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	34568 368	3456 <u>4</u>	7	13 <u>3</u>	145 1	2	458	4568 <u>5</u>	9
STEP 1	34568 68	3456 4	7	13 3	145 1	2	458 <u>8</u>	4568 5	9
STEP 2	34568 6	3456 4	7	13 3	145 1	2	458 8	4568 5	9

After updates of box D, box I, box F, row 4, col 8, and row 6:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	136 36	36	2	167 1	17 7	9	48	48 8	5
row2	146 1	8	46 4	5	3	16 6	9	7	2
row3	9	7	5	4	2	8	6	3	1
row4	2	34569 9	3469 36	8	459 5	347 7	1	456 4	367 36
row5	3458 5	1	3489 38	39 9	6	347 4	4578 7	2	378 38
row6	34568 6	3456 4	7	13 3	145 1	2	458 8	4568 5	9
row7	368	369 36	1	3679 7	79 9	5	2	689 6	4
row8	3456 4	2	3469 9	369 6	8	346 3	57 5	1	67 7
row9	7	4569 5	4689 68	2	149 4	146 1	3	5689 9	68

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

COLUMN 1-----			
	start	step1	step2
row1	136 36	136 <u>3</u>	136 3
row2	146 1	146 1	146 1
row3	9	9	9
row4	2	2	2
row5	3458 5	3458 5	3458 5
row6	34568 <u>6</u>	34568 6	34568 6
row7	368 38	368 38	368 8
row8	3456 4	3456 4	3456 4
row9	7	7	7

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	136 <u>3</u>	36	2	167 1	17 7	9	48	48 <u>8</u>	5
STEP 1	136 3	36 6	2	167 1	17 7	9	48 4	48 8	5

-----BOX D-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row4	2	34569 9	3469 36	2	34569 9	3469 <u>3</u>	2	34569 9	3469 3
row5	3458 5	1	3489 38	3458 5	1	3489 38	3458 5	1	3489 8
row6	34568 <u>6</u>	3456 4	7	34568 6	3456 4	7	34568 6	3456 4	7

-----BOX F-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row4	1	456 4	367 36	1	4	36	1	4	6
row5	4578 7	2	378 38	4578 7	2	<u>3</u>	4578 7	2	3
row6	458 <u>8</u>	4568 5	9	458 8	4568 5	9	458 8	4568 5	9

-----BOX G-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row7	368 <u>8</u>	369 36	1	368 8	369 36	1	368 8	369 3	1
row8	3456 4	2	3469 9	3456 4	2	3469 9	3456 4	2	3469 9
row9	7	4569 5	4689 68	7	4569 5	<u>6</u>	7	4569 5	4689 6

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	7	4569 5	4689 <u>6</u>	2	149 4	146 1	3	5689 9	68
STEP 1	7	4569 5	4689 6	2	149 4	146 1	3	5689 9	8

After updates of col 1, row 1, box D, box F, Box G, and row 9:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	136 3	36 6	2	167 1	17 7	9	48 4	48 8	5
row2	146 1	8	46 4	5	3	16 6	9	7	2
row3	9	7	5	4	2	8	6	3	1
row4	2	34569 9	3469 3	8	459 5	347 7	1	456 4	367 6
row5	3458 5	1	3489 8	39 9	6	347 4	4578 7	2	378 3
row6	34568 6	3456 4	7	13 3	145 1	2	458 8	4568 5	9
row7	368 8	369 3	1	3679 7	79 9	5	2	689 6	4
row8	3456 4	2	3469 9	369 6	8	346 3	57 5	1	67 7
row9	7	4569 5	4689 6	2	149 4	146 1	3	5689 9	68 8

All rows, columns, and boxes are 9-perfect, so subpattern-1 solves the sudoku.

Example I.

The following sudoku is shown after trapping, annotation and group separation; row-box, column-box and n-wing reduction; plus additional group separation after reduction:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467	3	468	24	9	68	12678	5	178
row2	2	589	45689	7	45	1	689	68	3
row3	5679	5789	1	23	235	68	4	27	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134	1347	34	1257	27	6
row6	157	6	25	8	17	9	1257	3	4
row7	4569	2589	7	1249	1245	45	3	168	1589
row8	3	589	4589	6	145	7	189	18	2
row9	569	1	2569	239	8	35	679	4	579

1	1		2			3		3			4			5
				2		3		3		4			5	
1					2	3		3			4			
							7					9		
6	6	6				8	8					9		
	6		7	7			8							

This is the 4-pattern. Box H has the most candidates, so we'll make it the starting box. We'll make the initial order of boxes **H→G→A→B→E**. With this order, all choices are forced for subpattern-1, beginning with square (74) as the first square of subpattern-1

A	B	
	E	
G	H	

→	↓	
	•	
↑	←	

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	4 ₁		4	4					
row2			4		4 ₁				
row3							X		
row4		X							
row5				4	4	4 ₁			
row6									X
row7	4			4 ₁	4	4			
row8			4 ₁		4				
row9								X	

We choose the beginning square of subpattern-2 to be (75), and use the same order of boxes as for subpattern-1, $H \rightarrow G \rightarrow A \rightarrow B \rightarrow E$. All choices are again forced, but we now have a contradiction in column 5, with two members of subpattern-2 in the same column. This means that square (75) cannot be a member of a subpattern, so we give it the killer K subscript, and try another square to begin subpattern-2. See the next page.

A	B	
	E	
G	H	

→	↓	
	•	
↑	←	

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	4 ₁₂		4	4					
row2			4		4 ₁₂				
row3							X		
row4		X							
row5				4	4	4 ₁			
row6									X
row7	4			4 ₁	4 ₂	4			
row8			4 ₁₂		4				
row9								X	

This time we choose square (76) as the beginning of subpattern-2, and, using the same order of boxes, have no problem with choices, as again they are all forced. $H \rightarrow G \rightarrow A \rightarrow B \rightarrow E$

A	B	
	E	
G	H	

→	↓	
	•	
↑	←	

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	4 ₁₂		4	4					
row2			4		4 ₁₂				
row3							X		
row4		X							
row5				4 ₂	4	4 ₁			
row6									X
row7	4			4 ₁	4 _K	4 ₂			
row8			4 ₁₂		4				
row9								X	

We choose the last available square, (85) as the beginning square for subpattern-3, but this time we notice there will be pending problems if we use the same order of boxes, so we change the box order to

$H \rightarrow B \rightarrow A \rightarrow G \rightarrow E$.

There are no pending situations, and all squares in the starting box have been accounted for, so there are no further subpatterns. The two unassigned candidates in squares, (13) and (55) must therefore be killer candidates, and we append to them the killer subscript K.

A	B	
G	H	

↓	←	
•	↑	

	B	
	E	
	H	

	↓	
	•	
	↑	

+

col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	4 ₁₂		4 _K	4 ₃				
row2			4 ₃		4 ₁₂			
row3						X		
row4		X						
row5				4 ₂	4 _K	4 ₁		
row6								X
row7	4 ₃			4 ₁	4 _K	4 ₂		
row8			4 ₁₂		4 ₃			
row9							X	

We first test subpattern-1, with 4's in squares (11), (25), (56), (74) and (83):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 4	3	468	24	9	68	12678	5	178
row2	2	589	45689	7	4	1	689	68	3
row3	5679	5789	1	23	235	68	4	27	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134	1347	34 4	1257	27	6
row6	157	6	25	8	17	9	1257	3	4
row7	4569	2589	7	1249 4	1245	45	3	168	1589
row8	3	589	4589 4	6	145	7	189	18	2
row9	569	1	2569	239	8	35	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX B-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row1	24	9	68	<u>2</u>	9	68	2	9	68
row2	7	45 <u>4</u>	1	7	45 4	1	7	45 4	1
row3	23	235	68	23	235	68	23 <u>3</u>	235 35	68

	step 3		
	col 4	col 5	col 6
row1	2	9	
row2	7	45 4	1
row3	23 3	235 5	68

-----BOX H-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row7	1249 <u>4</u>	1245	45	1249 4	1245 125	45 <u>5</u>	1249 4	1245 12	45 _5
row8	6	145	7	6	145 15	7	6	145 <u>1</u>	7
row9	239	8	35	239	8	35	239	8	35 <u>3</u>

	step 3			step 4		
	col 4	col 5	col 6	col 4	col 5	col 6
row7	1249 4	1245 <u>2</u>	45 _5	1249 4	1245 2	45 _5
row8	6	145 1	7	6	145 1	7
row9	239 29	8	35 3	239 9	8	35 3

COLUMN 5			
	start	step1	step2
row1	9	9	9
	45	45	45
row2	<u>4</u>	4	4
	235	235	235
row3	5	5	5
row4	6	6	6
	1347	1347	1347
row5		37	3
	17	17	17
row6		<u>7</u>	7
	1245	1245	1245
row7	2	2	2
	145	145	145
row8	<u>1</u>	1	1
row9	8	8	8

-----BOX E-----

start			
	col 4	col 5	col 6
row4	5	6	2
	134	1347	34
row5		<u>3</u>	<u>4</u>
		17	
row6	8	7	9

step 1			
	col 4	col 5	col 6
row4	5	6	2
	134	1347	34
row5	1	3	4
		17	
row6	8	7	9

This is the sudoku after updating by box B, box H, col 5 and box E:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 4	3	468	24 2	9	68	12678	5	178
row2	2	589	45689	7	45 4	1	689	68	3
row3	5679	5789	1	23 3	235 5	68	4	27	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134 1	1347 3	34 4	1257	27	6
row6	157	6	25	8	17 7	9	1257	3	4
row7	4569	2589	7	1249 4	1245 2	45 5	3	168	1589
row8	3	589	4589 4	6	145 1	7	189	18	2
row9	569	1	2569	239 9	8	35 3	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	3	589	4589	6	<u>1</u>	7	189	18	2
STEP 1	3	589	4589	6	1	7	189 89	<u>8</u>	2
STEP 2	3	589 59	4589	6	1	7	189 <u>9</u>	8	2
STEP 3	3	589 5	4589	6	1	7	189 9	8	2

-----BOX I-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row7	3	168	1589	3	168 16	1589 15
row8	189 <u>9</u>	18 <u>8</u>	2	189 9	18 8	2
row9	679	4	579	679 67	4	579 57

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	4569	2589	7	<u>4</u>	<u>2</u>	<u>5</u>	3	168 16	1589 15
STEP 1	4569 69	2589 89	7	4	2	5	3	168 16	1589 <u>1</u>
STEP 2	4569 69	2589 89	7	4	2	5	3	168 <u>6</u>	1589 1
STEP 3	4569 <u>9</u>	2589 89	7	4	2	5	3	168 6	1589 1
STEP 4	4569 9	2589 8	7	4	2	5	3	168 6	1589 1

-----BOX G-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row7	4569 <u>9</u>	2589 8	7	4569 9	2589 8	7	4569 9	2589 8	7
row8	3	589 <u>5</u>	4589 4	3	589 5	4589 4	3	589 5	4589 4
row9	569	1	2569	569 <u>6</u>	1	2569 26	569 6	1	2569 2

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	569 <u>6</u>	1	2569 2	239 9	8	35 3	679 67	4	579 57
STEP 1	569 6	1	2569 2	239 9	8	35 3	679 <u>7</u>	4	579 57
STEP 2	569 6	1	2569 2	239 9	8	35 3	679 7	4	579 5

This is the sudoku after updating by row 8, box I, row 7, box G and row 9:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 4	3	468	24 2	9	68	12678	5	178
row2	2	589	45689	7	45 4	1	689	68	3
row3	5679	5789	1	23 3	235 5	68	4	27	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134 1	1347 3	34 4	1257	27	6
row6	157	6	25	8	17 7	9	1257	3	4
row7	4569 9	2589 8	7	1249 4	1245 2	45 5	3	168 6	1589 1
row8	3	589 5	4589 4	6	145 1	7	189 9	18 8	2
row9	569 6	1	2569 2	239 9	8	35 3	679 7	4	579 5

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

COLUMN 2				
	start	step1	step2	step3
row1	3 589	3 589	3 589	3 589
row2		<u>9</u> 5789	9 5789	9 5789
row3		79	<u>7</u> 5789	7 5789
row4	4 2579	4 2579	4 2579	4 2579
row5		279	27	2
row6	6 2589	6 2589	6 2589	6 2589
row7	<u>8</u> 589	8 589	8 589	8 589
row8	<u>5</u>	5	5	5
row9	1	1	1	1

-----BOX D-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row4	17	4	3	17	4	3	17	4	3
row5	8	<u>2</u>	259	8	2	59	8	2	9
row6	157	6	25	157	6	<u>5</u>	157 17	6	25 5

		col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6	START	157 17	6	25 <u>5</u>	8	17 <u>7</u>	9	1257	3	4
	STEP 1	157 <u>1</u>	6	25 5	8	17 7	9	1257 12	3	4
	STEP 2	157 1	6	25 5	8	17 7	9	1257 2	3	4

		col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5	START	8	2579 <u>2</u>	259 9	134 <u>1</u>	1347 3	34 4	1257	27	6
	STEP 1	8	2579 2	259 9	134 1	1347 3	34 4	1257 57	27 <u>7</u>	6
	STEP 2	8	2579 2	259 9	134 1	1347 3	34 4	1257 5	27 7	6

COLUMN 1			
	start	step1	step2
row1	467	467	467
	4	4	4
row2	2	2	2
	5679	5679	5679
row3	57	57	5
	17	17	17
row4		<u>7</u>	7
	8	8	8
row5	157	157	157
	<u>1</u>	1	1
row6	4569	4569	4569
	<u>9</u>	9	9
row7	3	3	3
	569	569	569
row8	<u>6</u>	6	6

This is the sudoku after updating by col 2, box D, row 6, row5 and col 1.
But now there are duplicate 5's in row 3, so subpattern-1 is invalid.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 4	3	468	24 2	9	68	12678	5	178
row2	2	9	45689	7	45 4	1	689	68	3
row3	5679 5	5789 7	1	23 3	235 5	68	4	27	789
row4	17 7	4	3	5	6	2	178	9	178
row5	8	2579 2	259 9	134 1	1347 3	34 4	1257 5	27 7	6
row6	157 1	6	25 5	8	17 7	9	1257 2	3	4
row7	4569 9	2589 8	7	1249 4	1245 2	45 5	3	168 6	1589 1
row8	3	589 5	4589 4	6	145 1	7	189 9	18 8	2
row9	569 6	1	2569 2	239 9	8	35 3	679 7	4	579 5

Subpattern-2 has 4's in squares (11), (25), (54), (76) and (83):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 4	3	468	24	9	68	12678	5	178
row2	2	589	45689	7	4	1	689	68	3
row3	5679	5789	1	23	235	68	4	27	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134 4	1347	34	1257	27	6
row6	157	6	25	8	17	9	1257	3	4
row7	4569	2589	7	1249	1245	45 4	3	168	1589
row8	3	589	4589 4	6	145	7	189	18	2
row9	569	1	2569	239	8	35	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX B-----

start

	col 4	col 5	col 6
row1	24	9	68
row2	7	45 <u>4</u>	1
row3	23	235	68

step 1

	col 4	col 5	col 6
	24	9	68
	<u>2</u>		
	7	45 4	1
	23	235	68

step 2

	col 4	col 5	col 6
	24	9	68
	2		
	7	45 4	1
	23	235 35	68
	<u>3</u>		

step 3

	col 4	col 5	col 6
row1	24	9	68
	2		
row2	7	45 4	1
row3	23	235	68
	3	5	

COLUMN 4			
	start	step1	step2
row1	24 <u>2</u>	24 2	24 2
row2	7	7	7
row3	23 <u>3</u>	23 3	23 3
row4	5	5	5
row5	134 <u>4</u>	134 4	134 4
row6	8	8	8
row7	1249	1249 19	1249 1
row8	6	6	6
row9	239	239 <u>9</u>	239 9

-----BOX H-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row7	1249 <u>1</u>	1245	45 <u>4</u>	1249 1	1245 25	45 4	1249 1	1245 2	45 4
row8	6	145	7	6	145 <u>5</u>	7	6	145 5	7
row9	239 9	8	35	239 9	8	35	239 9	8	35 3

This is the sudoku after updating by box B, col 4 and box H. But now we have a problem. The three squares (86), (76) and (96) must share two digits, 3 & 4. This is a contradiction. Therefore subpattern-2 is invalid.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 4	3	468	24 2	9	68	12678	5	178
row2	2	589	45689	7	45 4	1	689	68	3
row3	5679	5789	1	23 3	235 5	68	4	27	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134 4	1347	34	1257	27	6
row6	157	6	25	8	17	9	1257	3	4
row7	4569	2589	7	1249 1	1245 2	45 4	3	168	1589
row8	3	589	4589 4	6	145 5	7	189	18	2
row9	569	1	2569	239 9	8	35 3	679	4	579

By the process of elimination, subpattern-3 must be the correct one, but we must still use it to arrive at the solution. Subpattern-3 has 4's in squares (14), (23), (56), (71) and (85):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467	3	468	24	9	68	12678	5	178
row2	2	589	45689	7	45	1	689	68	3
row3	5679	5789	1	23	235	68	4	27	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134	1347	34	1257	27	6
row6	157	6	25	8	17	9	1257	3	4
row7	4569	2589	7	1249	1245	45	3	168	1589
row8	3	589	4589	6	145	7	189	18	2
row9	569	1	2569	239	8	35	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	467	3	468	24 <u>4</u>	9	68	12678	5	178
STEP 1	467 67	3	468 68	24 4	9	68	12678	5	178
STEP 2	467 <u>7</u>	3	468 68	24 4	9	68	12678 127	5	178 17
STEP 3	467 7	3	468 68	24 4	9	68	12678 12	5	178 <u>1</u>
STEP 4	467 7	3	468 68	24 4	9	68	12678 2	5	178 1

	start	step1	step2	step3
row1	467 <u>7</u>	467 7	467 7	467 7
row2	2	2	2	2
row3	5679	5679 569	5679 569	5679 69
row4	17	17 <u>1</u>	17 1	17 1
row5	8	8	8	8
row6	157	157 15	157 <u>5</u>	157 5
row7	4569 4	4569 4	4569 4	4569 4
row8	3	3	3	3
row9	569	569	569	569 69

-----BOX D-----

start			
	col 1	col 2	col 3
row4	17 1	4	3
row5	8	2579	259
row6	157 <u>5</u>	6	25

step 1			
	col 1	col 2	col 3
row4	17 1	4	3
row5	8	2579 279	259 29
row6	157 5	6	25 <u>2</u>

step 2			
	col 1	col 2	col 3
row4	17 1	4	3
row5	8	2579 79	259 <u>9</u>
row6	157 5	6	25 2

step 3			
	col 1	col 2	col 3
row4	17 1	4	3
row5	8	2579 7	259 9
row6	157 5	6	25 2

		col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5	START	8	2579 <u>7</u>	259 9	134	1347	34 <u>4</u>	1257	27	6
	STEP 1	8	2579 7	259 9	134 <u>13</u>	1347 <u>13</u>	34 4	1257 125	27 <u>2</u>	6
	STEP 2	8	2579 7	259 9	134 13	1347 13	34 4	1257 5	27 2	6
		col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4	START	17 <u>1</u>	4	3	5	6	2	178	9	178
	STEP 1	17 1	4	3	5	6	2	178 78	9	178 78

After updating with row 1, col 1, box D, row 5, and row 4:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 7	3	468 68	24 4	9	68	12678 2	5	178 1
row2	2	589	45689 4	7	45	1	689 68	3	
row3	5679 69	5789	1	23	235	68	4	27	789
row4	17 1	4	3	5	6	2	178 78	9	178 78
row5	8	2579 7	259 9	134 13	1347 13	34 4	1257 5	27 2	6
row6	157 5	6	25 2	8	17	9	1257	3	4
row7	4569 4	2589	7	1249	1245	45	3	168	1589
row8	3	589	4589	6	145 4	7	189	18	2
row9	569 69	1	2569	239	8	35	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX F-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row4	178 <u>78</u>	9	178 <u>78</u>	178 78	9	178 78
row5	1257 <u>5</u>	27 <u>2</u>	6	1257 5	27 2	6
row6	1257	3	4	1257 1	3	4

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	157 5	6	25 2	8	17	9	1257 <u>1</u>	3	4
STEP 1	157 5	6	25 2	8	7	9	1257 1	3	4

-----BOX H-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row7	1249	1245	45	1249 129	1245 125	45 <u>5</u>	1249 129	1245 12	45 5
row8	6	145 <u>4</u>	7	6	145 4	7	6	145 4	7
row9	239	8	35	239	8	35	239	8	<u>3</u>

	step 3		
	col 4	col 5	col 6
row7	1249 129	1245 12	45 5
row8	6	145 4	7
row9	239 29	8	35 3

-----BOX A-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row1	467 <u>7</u>	3	468 68	467 7	3	468 68
row2	2	589	45689 4	2	589	45689 4
row3	5679 69	5789	1	5679 69	5789 589	1

-----BOX B-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row1	24 <u>4</u>	9	68	24 4	9	68	24 4	9	68
row2	7	45	1	7	45 <u>5</u>	1	7	45 5	1
row3	23	235	68	23	235	68	23	235 23	68

After updating with box F, row 6, box H, box A, and box B:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 7	3	468 68	24 4	9	68	12678 2	5	178 1
row2	2	589	45689 4	7	45 5	1	689 68	68	3
row3	5679 69	5789 589	1	23	235 23	68	4	27	789
row4	17 1	4	3	5	6	2	178 78	9	178 78
row5	8	2579 7	259 9	134 13	1347 13	34 4	1257 5	27 2	6
row6	157 5	6	25 2	8	17 7	9	1257 1	3	4
row7	4569 4	2589	7	1249 129	1245 12	45 5	3	168	1589
row8	3	589	4589	6	145 4	7	189	18	2
row9	569 69	1	2569	239 29	8	35 3	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	5679 69	5789 589	1	<u>23</u>	235 <u>23</u>	68	4	27	789
STEP 1	5679 69	5789 589	1	23	235 23	68	4	27 <u>7</u>	789
STEP 2	5679 <u>69</u>	5789 589	1	23	235 23	<u>68</u>	4	27 7	789 <u>89</u>
STEP 3	5679 69	5789 5	1	23	235 23	68	4	27 7	789 89

	start	step1	step2
row1	3	3	3
row2	589	589 <u>89</u>	589 89
row3	5789 <u>5</u>	5789 5	5789 5
row4	4	4	4
row5	2579 7	2579 7	2579 7
row6	6	6	6
row7	2589	2589 289	2589 2
row8	589	589 <u>89</u>	589 89
row9	1	1	1

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	4569 4	2589 <u>2</u>	7	1249 129	1245 12	45 <u>5</u>	3	168	1589
STEP 1	4569 4	2589 2	7	1249 19	1245 <u>1</u>	45 5	3	168	1589 189
STEP 2	4569 4	2589 2	7	1249 <u>9</u>	1245 1	45 5	3	168 68	1589 89
STEP 3	4569 4	2589 2	7	1249 9	1245 1	45 5	3	168 68	1589 <u>8</u>
STEP 4	4569 4	2589 2	4569 4	1249 9	1245 1	45 5	3	168 6	1589 8

-----COLUMN 9-----

	start	step1	step2
row1	178 1	178 1	178 1
row2	3	3	3
row3	789 89	789 <u>9</u>	789 9
row4	178 78	178 <u>7</u>	178 7
row5	6	6	6
row6	4	4	4
row7	1589 <u>8</u>	1589 8	1589 8
row8	2	2	2
row9	579	579	579 5

-----BOX C-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row1	12678 2	5	178	12678 2	5	178
row2	689	68	3	689 68	68	3
row3	4	27 7	789 <u>9</u>	4	27 7	789 9

After updating with row 3, col 2, row 7, col 9, and box C:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 7	3	468 68	24 4	9	68	12678 2	5	178 1
row2	2	589 89	45689 4	7	45 5	1	689	68	3
row3	5679 69	5789 5	1	23	235 23	68	4	27 7	789 9
row4	17 1	4	3	5	6	2	178 78	9	178 7
row5	8	2579 7	259 9	134 13	1347 13	34 4	1257 5	27 2	6
row6	157 5	6	25 2	8	17 7	9	1257 1	3	4
row7	4569 4	2589 2	7	1249 9	1245 1	45 5	3	168 6	1589 8
row8	3	589 89	4589	6	145 4	7	189	18	2
row9	569 69	1	2569	239 29	8	35 3	679	4	579 5

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

COLUMN 8			
	start	step1	step2
row1	5	5	5
	68	68	68
row2		<u>8</u>	8
	27	27	27
row3	7	7	7
row4	9	9	9
	27	27	27
row5	2	2	2
row6	3	3	3
	168	168	168
row7	<u>6</u>	6	6
	18	18	18
row8			1
row9	4	4	4

-----BOX I-----

start

	col 7	col 8	col 9
row7	3	168 <u>6</u>	1589 <u>8</u>
	189	18	
row8		<u>1</u>	2
	679		579
row9		4	5

step 1

	col 7	col 8	col 9
row7	3	168 6	1589 8
	189	18	
row8		<u>9</u>	1
	679		579
row9		79 4	5

step 2

	col 7	col 8	col 9
row7	3	168 6	1589 8
	189	18	
row8		9	1
	679		579
row9		7	4

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	5679 69	5789 5	1	23	235 23	68	4	27 7	789 <u>9</u>
STEP 1	5679 <u>6</u>	5789 5	1	23	235 23	68	4	27 7	789 9
STEP 2	5679 6	5789 5	1	23	235 23	68 8	4	27 7	789 9

COLUMN 6

	start	step1
row1	68	68 6
row2	1	1
row3	68 <u>8</u>	68 8
row4	2	2
row5	34 4	34 4
row6	9	9
row7	45 5	45 5
row8	7	7
row9	35 3	35 3

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	467 7	3	468 68	24 4	9	68 <u>6</u>	12678 2	5	178 1
STEP 1	467 7	3	468 8	24 4	9	68 6	12678 2	5	178 1

After updating with col 8, box I, row 3, col 6, and row 1:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 7	3	468 8	24 4	9	68 6	12678 2	5	178 1
row2	2	589 89	45689 4	7	45 5	1	689 68	8	3
row3	5679 6	5789 5	1	23	235 23	68 8	4	27 7	789 9
row4	17 1	4	3	5	6	2	178 78	9	178 7
row5	8	2579 7	259 9	134 13	1347 13	34 4	1257 5	27 2	6
row6	157 5	6	25 2	8	17 7	9	1257 1	3	4
row7	4569 4	2589 2	7	1249 9	1245 1	45 5	3	168 6	1589 8
row8	3	589 89	4589	6	145 4	7	189 9	18 1	2
row9	569 69	1	2569	239 29	8	35 3	679 7	4	579 5

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX A-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row1	467 7	3	468 <u>8</u>	467 7	3	468 8
row2	2	589	45689 4	2	589 9	45689 4
row3	5679 6	5789 <u>5</u>	1	5679 6	5789 5	1

COLUMN 2

	start	step1
row1	3	3
row2	589 <u>9</u>	589 9
row3	5789 <u>5</u>	5789 5
row4	4	4
row5	2579 7	2579 7
row6	6	6
row7	2589 2	2589 2
row8	589	589 8
row9	1	1

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	3	589 <u>8</u>	4589	6	145 <u>4</u>	7	189 <u>9</u>	18 1	2

STEP 1	3	589 8	4589 5	6	145 4	7	189 9	18 1	2
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COLUMN 1			-----BOX G-----			
	start	step1		col 1	col 2	col 3
row1	467	467	row7	4569	2589	
	7	7		4	<u>2</u>	7
row2	2	2	row8		589	4589
row3	5679	5679	row9	569		2569
	<u>6</u>	6		<u>9</u>	1	
row4	17	17				
	1	1				
row5	8	8				
row6	157	157	row7	4569	2589	
	5	5		4	2	7
row7	4569	4569	row8		589	4589
	4	4		3	8	5
row8			row9	569		2569
	3	3		9	1	6
row9	569 69	569 9				

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	569 <u>9</u>	1	2569 6	239 29	8	35 3	679 7	4	579 5

STEP 1	569 9	1	2569 6	239 2	8	35 3	679 7	4	579 5
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This is the sudoku after updating by box A, col 2, row 8, box G and row 9:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 7	3	468 8	24 4	9	68 6	12678 2	5	178 1
row2	2	589 9	45689 4	7	45 5	1	689 68	8	3
row3	5679 6	5789 5	1	23	235 23	68 8	4	27 7	789 9
row4	17 1	4	3	5	6	2	178 78	9	178 7
row5	8	2579 7	259 9	134 13	1347 13	34 4	1257 5	27 2	6
row6	157 5	6	25 2	8	17 7	9	1257 1	3	4
row7	4569 4	2589 2	7	1249 9	1245 1	45 5	3	168 6	1589 8
row8	3	589 8	4589 5	6	145 4	7	189 9	18 1	2
row9	569 9	1	2569 6	239 2	8	35 3	679 7	4	579 5

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

COLUMN 4

	start	step1	step2
row1	24 4	24 4	24 4
row2	7	7	7
row3	23	23 <u>3</u>	23 3
row4	5	5	5
row5	134 13	134 13	134 1
row6	8	8	8
row7	1249 9	1249 9	1249 9
row8	6	6	6
row9	239 <u>2</u>	239 2	239 2

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	8	2579 7	259 9	134 <u>1</u>	1347 13	34 4	1257 5	27 2	6

STEP 1	8	2579 7	259 9	134 1	1347 3	34 4	1257 5	27 2	6
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	2	589 <u>9</u>	45689 4	7	45 5	1	689	68 <u>8</u>	3

STEP 1	2	589 9	45689 4	7	45 5	1	689 6	68 8	3
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-----BOX F-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row4	178 78	9	178 <u>7</u>	178 8	9	178 7
row5	1257 5	27 2	6	1257 5	27 2	6
row6	1257 1	3	4	1257 1	3	4

After updating with col 4, row 5, row 2, and box F, all rows, columns, and boxes are 9-perfect, so subpattern-3 solves the sudoku.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 7	3	468 8	24 4	9	68 6	12678 2	5	178 1
row2	2	589 9	45689 4	7	45 5	1	689 6	68 8	3
row3	5679 6	5789 5	1	23 3	235 2	68 8	4	27 7	789 9
row4	17 1	4	3	5	6	2	178 8	9	178 7
row5	8	2579 7	259 9	134 1	1347 3	34 4	1257 5	27 2	6
row6	157 5	6	25 2	8	17 7	9	1257 1	3	4
row7	4569 4	2589 2	7	1249 9	1245 1	45 5	3	168 6	1589 8
row8	3	589 8	4589 5	6	145 4	7	189 9	18 1	2
row9	569 9	1	2569 6	239 2	8	35 3	679 7	4	579 5

Example J.

The following sudoku is shown after trapping, annotation and group separation; row-box, column-box and n-wing reduction; plus additional group separation after reduction:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	578	157	2	5678	17	4	9	67	3
row2	13578	6	1358	1578	9	578	18	2	4
row3	9	4	18	2	3	678	5	167	68
row4	4	2	15	3	8	69	7	16	69
row5	1378	9	138	67	5	2	138	4	68
row6	3578	57	6	79	4	1	38	59	2
row7	6	58	7	589	2	3	4	59	1
row8	15	3	4	1579	6	579	2	8	579
row9	2	158	9	4	17	578	6	3	57

			2	2	2		3	3	4	4	4			5
	1		2	2	2		3		4	4	4		5	
		1	2	2	2	3		3	4	4	4			

6							9	9	9
6						8	9		
6	6	6	7			8	9		

This is the 1-patten. We'll choose box A as the starting box, as it contains the largest number of candidates, and begin the first pattern in square (12). The best order of boxes is clearly **A→B→C→F→D→G→H**, as all the choices are forced.

A	B	C
D		F
G	H	

→	→	↓
↓		←
→	•	

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		1 ₁			1				
row2	1		1	1 ₁			1		
row3			1					1 ₁	
row4			1 ₁					1	
row5	1		1				1 ₁		
row6						X			
row7									X
row8	1 ₁			1					
row9		1			1 ₁				

We'll arbitrarily begin subpattern-2 in square (21). The same box order will be chosen for subpattern-2, as we note that all choices are again forced. The box order is **A→B→C→F→D→G→H**

A	B	C
D		F
G	H	

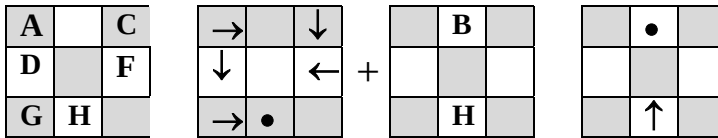
→	→	↓
↓		←
→	•	

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		1 ₁			1 ₂				
row2	1 ₂		1	1 ₁			1		
row3			1					1 ₂	
row4			1 ₁₂					1	
row5	1		1				1 ₂		
row6						X			
row7									X
row8	1 ₁			1 ₂					
row9		1 ₂			1 ₁				

Subpattern-3 is arbitrarily chosen to begin in square (23), and we try following the same route as before, $A \rightarrow B \rightarrow C \rightarrow F \rightarrow D \rightarrow G \rightarrow H$. Extensions are forced in boxes B, C and F, but when we get to box D, even though the choice is forced to be square (43), this creates two members of subpattern-3 to be in the same column, squares (43) and (23). Since all choices were forced, this means that square (23) in box A cannot be a member of a subpattern. Therefore it is a killer candidate. We must mark it with the subscript K, remove the subscript 2 from all the squares it was assigned to, and begin subpattern-3 in square (33), the only remaining available square in box A. We shall continue the development of subpattern-3 on the next page.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		1 ₁			1 ₂₃				
row2	1 ₂		1 _K	1 ₁			1		
row3			1					1 ₂₃	
row4			1 ₁₂					1	
row5	1		1				1 ₂₃		
row6						X			
row7									X
row8	1 ₁			1 ₂					
row9		1 ₂			1 ₁				

In determining subpattern-3, we must change the order of boxes because the continuation of subpattern-3 cannot be made immediately in box B. We'll change the box order to **A→C→F→D→G→H→B**:



At the conclusion of subpattern-3, there are no squares left in the starting box with which to begin any further subpatterns. Square (53) has not been assigned to a subpattern, so it must house a killer candidate.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1		1 ₁			1 ₂₃				
row2	1 ₂		1 _K	1 ₁			1 ₃		
row3			1 ₃					1 ₁₂	
row4			1 ₁₂					1 ₃	
row5	1 ₃		1 _K				1 ₂		
row6						X			
row7									X
row8	1 ₁			1 ₂₃					
row9		1 ₂₃			1 ₁				

We shall first test subpattern-1, with 1's in squares (12), (24), (38), (43), (57), (81), and (95).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	578	1	2	5678	17	4	9	67	3
row2	13578	6	1358	1578	1	9	578	18	2
row3	9	4	18	2	3	678	5	167	68
row4	4	2	1	3	8	69	7	16	69
row5	1378	9	138	67	5	2	1	4	68
row6	3578	57	6	79	4	1	38	59	2
row7	6	58	7	589	2	3	4	59	1
row8	15	3	4	1579	6	579	2	8	579
row9	2	158	9	4	17	578	6	3	57

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-BOX A

step 2		
col 1	col 2	col 3
578 57	157 1	2
13578 357	6	1358 35
9	4	18 8

BOX B.

step 2		
col 4	col 5	col 6
5678 568	17 7	4
1578 1	9	578 58
2	3	678 68

-BOX C.

step 2		
col 7	col 8	col 9
9	67	3
18 8	2	4
5	167 1	68 <u>6</u>

step 3

	col 7	col 8	col 9
row1	9	67 7	3
row2	18 8	2	4
row3	5	167 1	68 6

After updating by box A, box B and box C, there are two sevens in row 1.
As a consequence, subpattern-1 fails.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	578 57	157 1	2	5678 568	17 7	4	9	67 7	3
row2	13578 357	6	1358 35	1578 1	9	578 58	18 8	2	4
row3	9	4	18 8	2	3	678 68	5	167 1	68 6
row4	4	2	15 1	3	8	69	7	16	69
row5	1378	9	138	67	5	2	138 1	4	68
row6	3578	57	6	79	4	1	38	59	2
row7	6	58	7	589	2	3	4	59	1
row8	15 1	3	4	1579	6	579	2	8	579
row9	2	158	9	4	17 1	578	6	3	57

We'll next test subpattern-2, with 1's in squares (15), (21), (38), (43), (57), (84), and (92).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	578	157	2	5678	17	4	9	67	3
row2	13578	6	1358	1578	9	578	18	2	4
row3	9	4	18	2	3	678	5	167	68
row4	4	2	15	3	8	69	7	16	69
row5	1378	9	138	67	5	2	138	4	68
row6	3578	57	6	79	4	1	38	59	2
row7	6	58	7	589	2	3	4	59	1
row8	15	3	4	1579	6	579	2	8	579
row9	2	158	9	4	17	578	6	3	57

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX A-----

start

	col 1	col 2	col 3
row1	578	157	2
row2	13578 <u>1</u>	6	1358
row3	9	4	18

step 1

	col 1	col 2	col 3
row1	578	57	2
row2	13578 1	6	1358 358
row3	9	4	18 <u>8</u>

step 2

	col 1	col 2	col 3
row1	578 <u>57</u>	<u>57</u>	2
row2	13578 1	6	1358 35
row3	9	4	18 8

step 3

	col 1	col 2	col 3
row1	578 57	57	2
row2	13578 1	6	1358 3
row3	9	4	18 8

-----BOX C-----

start

	col 4	col 5	col 6
row1	9	67	3
row2	18	2	4
row3	5	167 <u>1</u>	68

step 1

	col 4	col 5	col 6
row1	9	67	3
row2	18	<u>8</u>	2
row3	5	1	68

step 2

	col 4	col 5	col 6
row1	9	67	3
row2	18	8	2
row3	5	1	68 <u>6</u>

step 3

	col 4	col 5	col 6
row1	9	7	3
row2	18 8	2	4
row3	5	1	68 6

After updating by box A and box C, we notice that there are **three** squares in row 1 which contain only the **two** digits 5 and 7. This is a contradiction, so subpattern-2 is also erroneous. This leaves only subpattern-3

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	578 57	157 57	2	5678	17 1	4	9	67 7	3
row2	13578 1	6	1358 3	1578	9	578	18 8	2	4
row3	9	4	18 8	2	3	678	5	167 1	68 6
row4	4	2	15 1	3	8	69	7	16	69
row5	1378	9	138	67	5	2	138 1	4	68
row6	3578	57	6	79	4	1	38	59	2
row7	6	58	7	589	2	3	4	59	1
row8	15	3	4	1579 1	6	579	2	8	579
row9	2	158 1	9	4	17	578	6	3	57

By default, subpattern-3, below, is the correct one, with 1's in squares (15), (27), (33), (48), (51), (84), and (92). We still need to go through the updating process with subpattern-3 to arrive at the solution.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	578	157	2	5678	17	4	9	67	3
row2	13578	6	1358	1578	9	578	18	1	2
row3	9	4	18	2	3	678	5	167	68
row4	4	2	15	3	8	69	7	16	69
row5	1378	9	138	67	5	2	138	4	68
row6	3578	57	6	79	4	1	38	59	2
row7	6	58	7	589	2	3	4	59	1
row8	15	3	4	1579	6	579	2	8	579
row9	2	158	9	4	17	578	6	3	57

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX D-----

start

	col 1	col 2	col 3
row4	4	2	15
row5	1378 <u>1</u>	9	138
row6	3578	57	6

step 1

	col 1	col 2	col 3
row4	4	2	<u>5</u>
row5	1378 1	9	138 38
row6	3578	57	6

step 2

	col 1	col 2	col 3
row4	4	2	5
row5	1378 1	9	138 38
row6	3578 378	57 <u>7</u>	6

step 3

	col 1	col 2	col 3
row4	4	2	5
row5	1378 1	9	138 38
row6	3578 38	57 7	6

-----BOX F-----

start

	col 7	col 8	col 9
row4	7	16 <u>1</u>	69
row5	138	4	68
row6	38	59	2

step 1

	col 7	col 8	col 9
row4	7	1	69
row5	138 <u>38</u>	4	68
row6	<u>38</u>	59	2

step 2

	col 7	col 8	col 9
row4	7	1	69
row5	138 38	4	68 <u>6</u>
row6	38	59	2

step 3

	col 7	col 8	col 9
row4	7	1	<u>9</u>
row5	138 38	4	68 6
row6	38	59	2

step 4

	col 7	col 8	col 9
row4	7	1	9
row5	138 38	4	68 6
row6	38	59 5	2

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	3578 38	57 <u>7</u>	6	79	4	1	38	59 5	2

STEP 1	3578 38	57 7	6	79 9	4	1	38	59 5	2
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-----BOX E-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row4	3	8	69	3	8	<u>6</u>	3	8	6
row5	67	5	2	67	5	2	67	5	2
row6	79 <u>9</u>	4	1	79 9	4	1	79 9	4	1

COLUMN 8

	start	step1
row1	67	67
row2	2	2
row3	167	167 67
row4	16 <u>1</u>	16 1
row5	4	4
row6	59 <u>5</u>	59 5
row7	59	59 9
row8	8	8
row9	3	3

This is the sudoku after updating by box D, box F, row 6, box E and col 8:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	578	157	2	5678	17		9	67	3
row2	13578	6	1358	1578	9	578	18	2	4
row3	9	4	18	2	3	678	5	167 67	68
row4	4	2	15	3	8	69	7	16	69
row5	1378		138 38	67 7	5	2	138 38	4	68
row6	3578 38	57 7	6	79 9	4	1	38	59 5	2
row7	6	58	7	589	2	3	4	59 9	1
row8	15	3	4	1579 1	6	579	2	8	579
row9	2	158 1	9	4	17	578	6	3	57

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	6	58	7	589	2	3	4	<u>9</u>	1

STEP 1	6	58	7	589 58	2	3	4	59 9	1
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-----BOX H-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row7	589 58	2	3	589 58	2	3	589 <u>58</u>	2	3
row8	1579 <u>1</u>	6		1579 1	6		1579 1	6	579 <u>59</u>
row9	4	17	578	4	17 <u>7</u>	578	4	17 7	578 <u>58</u>

	step 3		
	col 4	col 5	col 6
row7	589 58	2	3
row8	1579 1	6	579 <u>9</u>
row9	4	17 7	578 58

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	15 5	3	4	1579 <u>1</u>	6	579 <u>9</u>	2	8	579
STEP 1	15 <u>5</u>	3	4	1579 1	6	579 9	2	8	579 57
STEP 2	15 5	3	4	1579 1	6	579 9	2	8	579 7
	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	2	158 1	9	4	17 <u>7</u>	578 58	6	3	57
STEP 1	2	158 1	9	4	17 7	578 58	6	3	57 <u>5</u>
STEP 2	2	158 1	9	4	17 7	578 8	6	3	57 5

	-----COLUMN 6-----		
	start	step1	step2
row1	4 578	4 578	4 578
row2	678	57	5
row3		678	678
		7	7
row4	69 <u>6</u>	69 6	69 6
row5			
row6	2	2	2
row7			
row8	1	1	1
row9	3 579	3 579	3 579
	9	9	9
	578	578	578
	<u>8</u>	8	8

-----BOX B-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row1	5678	17		5678	17		5678	17	
		<u>1</u>	4	68	1	4	6	1	4
row2	1578		578	1578		578	1578		578
		9	<u>5</u>	8	9	5	8	9	5
row3			678			678			678
	2	3	<u>7</u>	2	3	7	2	3	7

This is the sudoku after updating with row 7, box H, row 8, row 9, col 6 and box B:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	578	157	2	5678	17	4	9	67	3
row2	13578	6	1358	1578	9	578	18	2	4
row3	9	4	18	2	3	678	5	167	68
row4	4	2	15	3	8	69	7	16	69
row5	1378	9	138	67	5	2	138	4	68
row6	3578	57	6	79	4	1	38	59	2
row7	6	58	7	589	2	3	4	59	1
row8	15	3	4	1579	6	579	2	8	579
row9	2	158	9	4	17	578	6	3	57

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	13578 	6	1358	1578 <u>8</u>	9	578 <u>5</u>	18 <u>1</u>	2	4
STEP 1	13578 37	6	1358 <u>3</u>	1578 8	9	578 5	18 1	2	4
STEP 2	13578 7	6	1358 3	1578 8	9	578 5	18 1	2	4

-----BOX A-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	578 	157	2	578 58	<u>5</u>	2	578 8	5	2
row2	13578 <u>7</u>	6	1358 3	13578 7	6	1358 3	13578 7	6	1358 3
row3	9	4	18 <u>1</u>	9	4	18 1	9	4	18 1

	COLUMN 1		COLUMN 2		COLUMN 3	
	start	step1	start	step1	start	step1
row1	578 <u>8</u>	578 8	157 <u>5</u>	157 5	2	2
row2	13578 7	13578 7	6	6	1358 <u>3</u>	1358 3
row3	9	9	4	4	18 1	18 1
row4	4	4	2	2	15 5	15 5
row5	1378 1	1378 1	9	9	138 38	138 8
row6	3578 38	3578 3	57 7	57 7	6	6
row7	6	6	58	58 8	7	7
row8	15 5	15 5	3	3	4	4
row9	2	2	158 1	158 1	9	9

This is the sudoku after further updating with row 2, box A, col 1, col 2 and col 3:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	578 8	157 5	2	5678 6	17 1	4	9	67	3
row2	13578 7	6	1358 3	1578 8	9	578 5	18 1	2	4
row3	9	4	18 1	2	3	678 7	5	167 67	68
row4	4	2	15 5	3	8	69 6	7	16 1	69 9
row5	1378 1	9	138 8	67 7	5	2	138 38	4	68 6
row6	3578 3	57 7	6	79 9	4	1	38	59 5	2
row7	6	58 8	7	589 58	2	3	4	59 9	1
row8	15 5	3	4	1579 1	6	579 9	2	8	579 7
row9	2	158 1	9	4	17 7	578 8	6	3	57 5

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	578 8	157 5	2	5678 <u>6</u>	17 1	4	9	67	3

STEP 1	578 8	157 5	2	5678 6	17 1	4	9	67 7	3
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-----BOX C-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row1	9	67 <u>7</u>	3	9	67 7	3	9	67 7	3
row2	18 1	2	4	18 1	2	4	18 1	2	4
row3	5	167 67	68	5	167 <u>6</u>	68	5	167 6	8

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	1378 1	9	138 <u>8</u>	67 7	5	2	138 38	4	68 6

STEP 1	1378 1	9	138 8	67 7	5	2	138 3	4	68 6
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	3578 <u>3</u>	57 7	6	79 9	4	1	38	59 5	2

STEP 1	3578 3	57 7	6	79 9	4	1	38 8	59 5	2
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-----BOX H-----

	start			step 1		
	col 4	col 5	col 6	col 4	col 5	col 6
row7	589 58	2	3	589 5	2	3
row8	1579 1	6	579 9	1579 1	6	579 9
row9	4	17 7	578 <u>8</u>	4	17 7	578 8

After final updates by row 1, box C, row 5, row 6 and box H, here is the solution. A check of all the rows, columns, and boxes shows them to be 9-perfect.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	578 8	157 5	2	5678 6	17 1	4	9	67 7	3
row2	13578 7	6	1358 3	1578 8	9	578 5	18 1	2	4
row3	9	4	18 1	2	3	678 7	5	167 6	68 8
row4	4	2	15 5	3	8	69 6	7	16 1	69 9
row5	1378 1	9	138 8	67 7	5	2	138 3	4	68 6
row6	3578 3	57 7	6	79 9	4	1	38 8	59 5	2
row7	6	58 8	7	589 5	2	3	4	59 9	1
row8	15 5	3	4	1579 1	6	579 9	2	8	579 7
row9	2	158 1	9	4	17 7	578 8	6	3	57 5

Example K.

The following sudoku is shown after trapping, annotation and group separation; row-box, column-box and n-wing reduction; plus additional group separation after reduction:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	7	16	124	9	24	146	8	3
row2	4	38	38	15	6	7	15	2	9
row3	16	9	2	1348	58	348	7	456	56
row4	137	1348	13789	6	24	5	389	39	27
row5	2	6	59	378	78	38	59	1	4
row6	37	3458	378	9	24	1	368	356	27
row7	1367	135	4	78	578	689	2	3569	56
row8	9	35	367	247	1	246	346	3456	8
row9	8	2	56	45	3	469	4569	7	1

			2		2			3	4			5		
	1	1	2								4		5	
	1	1	2		2		3		4					

	6		7	7	7			8	9	9	9
6	6									9	
					7	8		8	9		

Although the five pattern is the least likely one to choose with so few established squares, it turns out to have a useful pattern.

This is the 5-pattern. The large X's are the squares with established 5's. The 5's are the squares with 5-candidates.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	X								
row2				5			5		
row3					5			5	5
row4						X			
row5			5				5		
row6		5						5	
row7		5			5				5
row8		5						5	
row9			5	5			5		

In analyzing this pattern, there is the desire to avoid pending boxes, and to do so requires some art in the choice of box order. We'll choose box I as the starting box, and for the first three subpatterns the following box orders:

subpattern-1: **I→H→G→D→F→C→B**

subpattern-2: **I→F→D→G→H→B→C**

subpattern-3: **I→H→G→D→F→B→C**

On the next page, we'll begin by analyzing subpattern-1.

Subpattern-1: The order of boxes is **I→H→G→D→F→C→B**.

	B	C
D		F
G	H	I

	•	←
→		↑
↑	←	←

Note that at each step, there is only one square not in the neighborhood of the preceding choices for subpattern members. Each "choice" is forced, there being no other choice possible. Therefore there are no pending boxes. This means that we may use a different order of boxes for the next subpattern.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	X								
row2				5			5 ₁		
row3					5 ₁			5	5
row4						X			
row5			5 ₁				5		
row6		5						5 ₁	
row7		5			5				5 ₁
row8		5 ₁						5	
row9			5	5 ₁			5		

Subpattern-2: The order of boxes is **I→F→D→G→H→B→C**.

	B	C
D		F
G	H	I

	→	•
↓		←
→	↑	↑

Note that at each step, there is only one square not in the neighborhood of the preceding choices for subpattern members. Each "choice" is forced, there being no other choice possible. Therefore there are no pending boxes. This means that we may again use a different order of boxes for the next subpattern.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	X								
row2				5 ₂			5 ₁		
row3					5 ₁			5	5 ₂
row4						X			
row5			5 ₁				5 ₂		
row6		5 ₂						5 ₁	
row7		5			5 ₂				5 ₁
row8		5 ₁						5 ₂	
row9			5 ₂	5 ₁			5		

Subpattern-3: The order of boxes is chosen to be **I→H→G→D→F→C→B**.

	B	C
D		F
G	H	I

	•	←
→		↑
↑	←	←

Note that at each step, there is only one square not in the neighborhood of the preceding choices for subpattern members. Each "choice" is forced, there being no other choice possible. Therefore there are no pending boxes. This means that we may use a different order of boxes for the next subpattern.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	X								
row2				5 ₂₃			5 ₁		
row3					5 ₁			5	5 ₂₃
row4						X			
row5			5 ₁₃				5 ₂		
row6		5 ₂						5 ₁₃	
row7		5			5 ₂₃				5 ₁
row8		5 ₁₃						5 ₂	
row9			5 ₂	5 ₁			5 ₃		

Finally, there are no squares in the starting box I which are not members of subpatterns, and there are no pending boxes, so the analysis is complete. The candidates in squares (38) and (72) are not members of any subpattern, so they are killer candidates, and are given the subscript K as a reminder. They should be eliminated from these two squares as candidates, but this is not actually necessary in the process of subpattern evaluation, because in the testing of the subpatterns, they will naturally be eliminated, since each is in the same box as a bona fide 5-candidate.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	X								
row2				5 ₂			5 ₁		
row3					5 ₁			5 _K	5 ₂₃
row4						X			
row5			5 ₁₃				5 ₂		
row6		5 ₂						5 ₁₃	
row7		5 _K			5 ₂₃				5 ₁
row8		5 ₁₃						5 ₂	
row9			5 ₂	5 ₁			5 ₃		

We first test subpattern-1, with 5's in squares (27), (35), (53), (68), (79), (82), (94).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	7	16	124	9	24	146	8	3
row2	4	38	38	15	6	7	15	2	9
row3	16	9	2	1348	58	348	7	456	56
row4	137	1348	13789	6	24	5	389	39	27
row5	2	6	5	378	78	38	59	1	4
row6	37	3458	378	9	24	1	368	356	27
row7	1367	135	4	78	578	689	2	3569	56
row8	9	35	367	247	1	246	346	3456	8
row9	8	2	56	45	5	469	4569	7	1

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX B-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row1	124	9	24	124	9	24	124	9	<u>24</u>
row2	15	6	7	15	<u>1</u>	7	15	6	7
row3	1348	58	348	1348	58	348	1348	58	348
		<u>5</u>			5		<u>348</u>	5	

	step 3		
	col 4	col 2	col 3
row1	124 24	9	24
row2	15 1	6	7
row3	1348 38	58 5	348 38

-----BOX C-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row1	146	8	3	146	8	3	146	8	3
row2	15	2	9	15	2	9	15	2	9
row3	7	456	56	7	456	56	7	456	56
					<u>46</u>	<u>6</u>		<u>4</u>	6

	step 3		
	col 7	col 8	col 9
row1	146 1	8	3
row2	15 5	2	9
row3	7	456 4	56 6

-----BOX G-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row7	1367	135	4	1367	135 13	4	1367 137	135 13	4
row8	9	35 <u>5</u>	367	9	35 5	367	9	35 5	367 37
row9	8	2	56	8	2	56 <u>6</u>	8	2	56 6

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	5	7	16	124 24	9	24	146 <u>1</u>	8	3
STEP 1	5	7	16 6	124 24	9	24	146 1	8	3

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

See this partially updated sudoku on the next page.

This is the sudoku after updating BOX B, BOX C, BOX G and ROW 1.
Note the duplicate 6's in column 3. We need go no further with the updating of subpattern-1. It is clearly invalid..

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	7	¹⁶ 6	¹²⁴ 24	9	²⁴	¹⁴⁶ 1	8	3
row2	4	³⁸	³⁸	¹⁵ 1	6	7	¹⁵ 5	2	9
row3	¹⁶	9	2	¹³⁴⁸ 38	⁵⁸ 5	³⁴⁸ 38	7	⁴⁵⁶ 4	⁵⁶ 6
row4	¹³⁷	¹³⁴⁸	¹³⁷⁸⁹	6	²⁴	5	³⁸⁹	³⁹	²⁷
row5	2	6	⁵⁹ <u>5</u>	³⁷⁸	⁷⁸	³⁸	⁵⁹	1	4
row6	³⁷	³⁴⁵⁸	³⁷⁸	9	²⁴	1	³⁶⁸	³⁵⁶ <u>5</u>	²⁷
row7	¹³⁶⁷ 137	¹³⁵ 13	4	⁷⁸	⁵⁷⁸	⁶⁸⁹	2	³⁵⁶⁹	⁵⁶ <u>5</u>
row8	9	³⁵ 5	³⁶⁷ 37	²⁴⁷	1	²⁴⁶	³⁴⁶	³⁴⁵⁶	8
row9	8	2	⁵⁶ 6	⁴⁵ <u>5</u>	3	⁴⁶⁹	⁴⁵⁶⁹	7	1

We next test subpattern-2, with 5's in squares (24), (39), (57), (62), (75), (88), and (93).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	7	16	124	9	24	146	8	3
row2	4	38	38	15	6	7	15	2	9
row3	16	9	2	1348	58	348	7	456	56
row4	137	1348	13789	6	24	5	389	39	27
row5	2	6	59	378	78	38	59	5	1
row6	37	3458	378	9	24	1	368	356	27
row7	1367	135	4	78	578	689	2	3569	56
row8	9	35	367	247	1	246	346	3456	5
row9	8	2	56	45	3	469	4569	7	1

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX G-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row7	1367	135	4	1367	135 13	4	1367 167	135 <u>1</u>	4
row8	9	35	367	9	35	367	9	35	367 67
row9	8	2	56 <u>5</u>	8	2	56 5	8	2	56 5

step 3			-----COLUMN 2-----				
	col 1	col 2	col 3	start	step1	step2	step3
row7	1367 67	135 1	4	row1	7	7	7
row8	9	35 3	367 67	row2	38	38 <u>8</u>	38 8
row9	8	2	56 5	row3	9	9	9
				row4	1348	1348 148	1348 4
				row5	6	6	6
				row6	3458 <u>5</u>	3458 5	3458 5
				row7	135 13	135 <u>1</u>	135 1
				row8	35 <u>3</u>	35 3	35 3
				row9	2	2	2

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	4	38 <u>8</u>	38	15 <u>5</u>	6	7	15	2	9
STEP 1	4	38 8	38 3	15 5	6	7	15 1	2	9

-----BOX I-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row7	2	3569	56	2	3569 369	<u>6</u>	2	3569 39	6
row8	346	3456 <u>5</u>	8	346	3456 5	8	346 34	3456 5	8
row9	4569	7	1	4569 469	7	1	4569 49	7	1

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	1367 67	135 1	4	78	578 5	689	2	3569 39	56 <u>6</u>
STEP 1	1367 <u>7</u>	135 1	4	78	578 5	689 89	2	3569 39	56 6
STEP 2	1367 7	135 1	4	78 <u>8</u>	578 5	689 89	2	3569 39	56 6
STEP 3	1367 7	135 1	4	78 8	578 5	689 <u>9</u>	2	3569 39	56 6
STEP 4	1367 7	135 1	4	78 8	578 5	689 9	2	3569 3	56 6

After updating box G, col 2, row 2, box I, and row 7:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	7	16	124	9	24	146	8	3
row2	4	38 8	38 3	15 5	6	7	15 1	2	9
row3	16	9	2	1348	58	348	7	456	56 5
row4	137	1348 4	13789	6	24	5	389	39	27
row5	2	6	59	378	78	38	59 5	1	4
row6	37	3458 5	378	9	24	1	368	356	27
row7	1367 7	135 1	4	78 8	578 5	689 9	2	3569 3	56 6
row8	9	35 3	367 67	247	1	246	346 34	3456 5	8
row9	8	2	56 5	45	3	469	4569 49	7	1

The solved squares of the sudoku above are shown below in black:

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX H-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row7	78 8	578 <u>5</u>	689 <u>9</u>	78 8	578 5	689 9	78 8	578 5	689 9
row8	247	1	246	247	1	246	247 27	1	246 26
row9	45	3	469	45 <u>4</u>	3	469 46	45 4	3	469 <u>6</u>

	step 3			step 4		
	col 4	col 5	col 6	col 4	col 5	col 6
row7	78 8	578 5	689 9	78 8	578 5	689 9
row8	247 27	1	246 <u>2</u>	247 7	1	246 2
row9	45 4	3	469 6	45 4	3	469 6

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	9	35 <u>3</u>	367 67	247 <u>7</u>	1	246 2	346 34	3456 5	8
STEP 1	9	35 3	367 6	247 7	1	246 2	346 4	3456 5	8

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

(Note no change since last digit map because only diminutions to candidate lists.)

	-----COLUMN 7-----			-----COLUMN 3-----		
	start	step1	step2	start	step1	step5
row1	146	146 <u>6</u>	146 6	16	16 <u>1</u>	16 1
row2	15 <u>1</u>	15 1	15 1	38 <u>3</u>	38 3	38 3
row3	7	7	7	2	2	2
row4	389	389	389 38	13789	13789 1789	13789 78
row5	59 5	59 5	59 5	59	59 <u>9</u>	59 9
row6	368	368	368 38	378	378 78	378 78
row7	2	2	2	4	4	4
row8	346 <u>4</u>	346 4	346 4	367 <u>6</u>	367 6	367 6
row9	4569 49	4569 <u>9</u>	4569 9	56 <u>5</u>	56 5	56 5

After updates of box G, col 2, row 2, box I, row 7, box H, row 8, col 7 and col 3:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	7	16	124	9	24	146	8	3
row2	4	38 8	38 3	15 5	6	7	15 1	2	9
row3	16	9	2	1348	58	348	7	456	56 5
row4	137	1348 4	13789	6	24	5	389 38	39	27
row5	2	6	59	378	78	38	59 5	1	4
row6	37	3458 5	378	9	24	1	368 38	356	27
row7	1367 7	135 1	4	78 8	578 5	689 9	2	3569 3	56 6
row8	9	35 3	367 6	247 7	1	246 2	346 4	3456 5	8
row9	8	2	56 5	45 4	3	469 6	4569 9	7	1

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----**COLUMN 1**-----

	start	step1	step2	step3
row1	5	5	5	5
row2	4	4	4	4
row3	16	16	16	16 6
row4	137	137 13	137 <u>1</u>	137 1
row5	2	2	2	2
row6	37	37 <u>3</u>	37 3	37 3
row7	1367 <u>7</u>	1367 7	1367 7	1367 7
row8	9	9	9	9
row9	8	8	8	8

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	16 <u>6</u>	9	2	1348	58	348	7	456	56 <u>5</u>
STEP 1	16 6	9	2	1348	58 <u>8</u>	348	7	456 <u>4</u>	56 5
STEP 2	16 6	9	2	1348 13	58 8	348 <u>3</u>	7	456 4	56 5
STEP 3	16 6	9	2	1348 1	58 8	348 3	7	456 4	56 5

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	37 <u>3</u>	3458 <u>5</u>	378	9	24	1	368	356	27
STEP 1	37 3	3458 5	378 78	9	24	1	368 68	356 <u>6</u>	27
STEP 2	37 3	3458 5	378 78	9	24	1	368 <u>8</u>	356 6	27
STEP 3	37 3	3458 5	378 <u>7</u>	9	24	1	368 8	356 6	27
STEP 4	37 3	3458 5	378 7	9	24	1	368 8	356 6	27 <u>2</u>
STEP 5	37 3	3458 5	378 7	9	24 4	1	368 8	356 6	27 2

-----BOX F-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row4	389 38	39	27	389 <u>3</u>	39	27 7	389 3	39 9	27 7
row5	59 5	1	4	59 5	1	4	59 5	1	4
row6	368 <u>8</u>	356 6	27 <u>2</u>	368 8	356 6	27 2	368 8	356 6	27 2

After updating by col 1, row 3, row 6 and box F:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	7	16	124	9	24	146	8	3
row2	4	38	38	15	6	7	15	2	9
row3	6	9	2	1348	58	348	7	456	56
row4	137	1348	13789		24		389	39	27
row5	2	6	59	378	78	38	59	5	1
row6	37	3458	378		24		368	356	27
row7	1367	135		78	578	689		3569	56
row8	9	35	367	247		246	346	3456	
row9	8	2	56	45		469	4569		

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

---COLUMN 6-----			-----BOX E-----					
			START			STEP 1		
			col 4	col 5	col 6	col 4	col 5	col 6
row1	start	step1						
	24	24						
		4						
row2	7	7						
	348	348						
row3	<u>3</u>	3						
	5	5						
row4	38	38						
row5		8						
row6	1	1						
	689	689						
row7	9	9						
	246	246						
row8	<u>2</u>	2						
	469	469						
row9	6	6						

		col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1	START	5	7	16	124	9	24	146	8	3
								<u>6</u>		
	STEP 1	5	7	16	124	9	24	146	8	3
				<u>1</u>				6		
	STEP 2	5	7	16	124	9	24	146	8	3
				1	24			6		

-
--COLUMN 4--

	start	step1
row1	124 24	124 2
row2	15 5	15 5
row3	1348 1	1348 1
row4	6	6
row5	378	378 3
row6	9	9
row7	78 8	78 8
row8	247 7	247 7
row9	45 4	45 4

-----BOX B-----
start

	col 4	col 5	col 6
row1	124 <u>2</u>	9	24
row2	15 5	6	7
row3	1348 1	58 8	348 3

---BOX D--
start

	col 1	col 2	col 3
row4	137 <u>1</u>	1348 4	13789
row5	2	6	59
row6	37 <u>3</u>	3458 <u>5</u>	378 <u>7</u>

step 1

	col 4	col 5	col 6
row1	124 2	9	24 4
row2	15 5	6	7
row3	1348 1	58 8	348 3

step 1

	col 1	col 2	col 3
row4	137 1	1348 4	13789 89
row5	2	6	59 <u>9</u>
row6	37 3	3458 5	378 7

step 2

	col 1	col 2	col 3
row4	137 1	1348 4	13789 8
row5	2	6	59 9
row6	37 3	3458 5	378 7

After updates of col 6, box E, row 1, col 4, box B, and box D. Note that all rows, columns, and boxes are 9-perfect, so subpattern-2 is the correct subpattern and leads to the solution below:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	5	7	16 1	124 2	9	24 4	146 6	8	3
row2	4	38 8	38 3	15 5	6	7	15 1	2	9
row3	16 6	9	2	1348 1	58 8	348 3	7	456 4	56 5
row4	137 1	1348 4	13789 8	6	24 2	5	389 3	39 9	27 7
row5	2	6	59 9	378 3	137 1	1348 4	13789 8	1	4
row6	37 3	3458 5	378 7	9	2	6	59 9	356 6	27 2
row7	1367 7	135 1	4	78 8	37 3	3458 5	378 7	3569 3	56 6
row8	9	35 3	367 6	247 7	1	246 2	346 4	3456 5	8
row9	8	2	56 5	45 4	3	469 6	4569 9	7	1

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

Example L.

The following sudoku is shown after trapping, annotation and group separation; row-box, column-box and n-wing reduction; plus additional group separation after reduction:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467	3	468	24	9	68	12678	5	178
row2	2	589	45689	7	45	1	689	68	3
row3	5679	5789	1	23	235	68	4	2678	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134	1347	34	1257	127	6
row6	157	6	25	8	17	9	1257	3	4
row7	4569	2589	7	1249	1245	45	3	168	1589
row8	3	589	4589	6	145	7	189	18	2
row9	569	1	2569	239	8	35	679	4	579

1	1		2			3		3			4			5
				2		3		3		4			5	
1					2	3		3			4			
							7					9		
6	6	6				8	8					9		
	6		7	7			8							

The 1-, 2-, 5-, 7-, 8-, and 9-patterns look too complicated. The 3-pattern looks too simple. Offhand, the 4- and 6-patterns seem more likely to be useful. We'll choose the 4-pattern, which is shown on the next page.

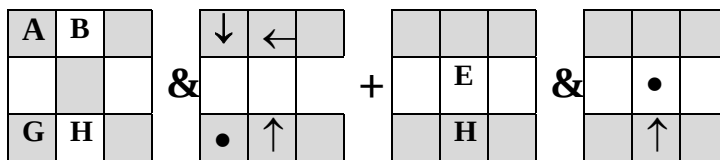
This is the 4-pattern. We'll choose box H as the starting box, since it has more candidates, and we'll choose the order of boxes for the first subpattern to be $H \rightarrow G \rightarrow A \rightarrow B \rightarrow E$. This sequence is chosen so as to avoid pending situations.

A	B	
	E	
G	H	

→	↓	
	•	
↑	←	

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	4 ₁		4	4					
row2			4		4 ₁				
row3							X		
row4		X							
row5				4	4	4 ₁			
row6									X
row7	4			4 ₁	4	4			
row8			4 ₁		4				
row9								X	

We'll choose the order of boxes for the second subpattern to be **H→B→A→G**, followed by **H→E**. This sequence is again chosen so as to avoid pending situations. However, when we compare G and H, we notice that the two members of subpattern-2 are in the same row. All the choices in this sequence were forced, so square (75) cannot be the start of any subpattern, and we give it the subscript K. We must also remove the subscript 2 from all the other squares it was assigned to. See the next page for subsequent operations.



	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	4 ₁		4	4 ₂					
row2			4 ₂		4 ₁				
row3							X		
row4		X							
row5				4	4	4 ₁			
row6									X
row7	4 ₂			4 ₁	4 ₂	4			
row8			4 ₁		4				
row9								X	

We'll start a new, subpattern-2 with square (76), and choose the sequence of boxes to be $H \rightarrow G \rightarrow A \rightarrow B \rightarrow E$.

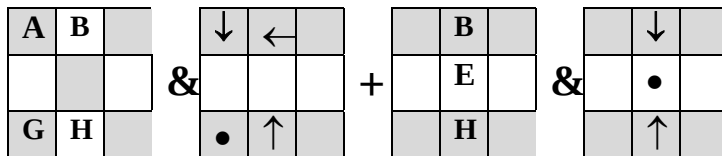
A	B	
	E	
G	H	

&

→	↓	
	•	
↑	←	

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	4 ₁₂		4	4					
row2			4		4 ₁₂				
row3							X		
row4		X							
row5				4 ₂	4	4 ₁			
row6									X
row7	4			4 ₁	4 _K	4 ₂			
row8			4 ₁₂		4				
row9								X	

We'll end with subpattern-3 starting in square (85) and choose the sequence of boxes to be **H→B→A→G**, followed by **H→E←B**. We note that squares (13) and (55) are not included in any subpattern, so they must both be killers, but each will be automatically excluded by its subpattern's digit.



This is the completed subpattern analysis of the 4-pattern:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	4 ₁₂		4 _K	4 ₃					
row2			4 ₃		4 ₁₂				
row3							X		
row4		X							
row5				4 ₂	4 _K	4 ₁₃			
row6									X
row7	4 ₃			4 ₁	4 _K	4 ₂			
row8			4 ₁₂		4 ₃				
row9								X	

We first test subpattern-1, with 4's in squares (11), (25), (56), (74) and (83):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 4	3	468	24	9	68	12678	5	178
row2	2	589	45689	7	4	1	689	68	3
row3	5679	5789	1	23	235	68	4	2678	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134	1347	34	1257	127	6
row6	157	6	25	8	17	9	1257	3	4
row7	4569	2589	7	1249	1245	45	3	168	1589
row8	3	589	4589	6	145	7	189	18	2
row9	569	1	2569	239	8	35	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX B-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row1	24	9	68	24	<u>2</u>	9	24	9	68
row2	7	45	1	7	45	1	7	45	1
row3	23	235	68	23	235	68	23	235	68

	step 3		
	col 4	col 5	col 6
row1	24	9	68
row2	7	45	1
row3	23	235	68

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1	467		468	24		68	12678		178
START	<u>4</u>	3		<u>2</u>	9			5	
STEP 1	467		468	24		<u>68</u>	12678		178
	4	3	<u>68</u>	2	9		1678	5	
STEP 2	467		468	24		68	12678		178
	4	3	68	2	9		17	5	17

This is the sudoku after updating by box B, row 1, col 4, box H and row 7:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 4	3	468 68	24 2	9	68	12678 17	5	178 17
row2	2	589	45689	7	45 4	1	689	68	3
row3	5679	5789	1	23 3	235 5	68	4	2678	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134 1	1347	34 4	1257	127	6
row6	157	6	25	8	17	9	1257	3	4
row7	4569 69	2589 89	7	1249 4	1245 2	45 5	3	168 68	1589 1
row8	3	589	4589 4	6	145 1	7	189	18	2
row9	569	1	2569	239 9	8	35 3	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX I-----

start

	col 7	col 8	col 9
row7	3	168 68	1589 <u>1</u>
row8	189	18	2
row9	679	4	579

step 1

	col 7	col 8	col 9
row7	3	168 68	1589 1
row8	189 89	18 <u>8</u>	2
row9	679	4	579

step 2

	col 7	col 8	col 9
row7	3	168 <u>6</u>	1589 1
row8	189 <u>9</u>	18 8	2
row9	679	4	579

step 3

	col 7	col 8	col 9
row7	3	6	1
row8	189 9	18 8	2
row9	679 <u>7</u>	4	579 57

step 4

	col 7	col 8	col 9
row7	3	6	1
row8	189 9	18 8	2
row9	679 7	4	579 5

But now we have a contradiction in squares (28), (78), and (88), because two digits (6 and 8) share three boxes. Subpattern-1 is therefore invalid.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 4	3	468 68	24 2	9	68	12678 17	5	178 17
row2	2	589	45689	7	45 4	1	689	68	3
row3	5679	5789	1	23 3	235 5	68	4	2678	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134 1	1347	34 4	1257	127	6
row6	157	6	25	8	17	9	1257	3	4
row7	4569 69	2589 89	7	1249 4	1245 2	45 5	3	168 6	1589 1
row8	3	589	4589 4	6	145 1	7	189 9	18 8	2
row9	569 6	1	2569 2	239 9	8	35 3	679 7	4	579 5

Subpattern-2 is composed of 4's in squares (11), (25), (54), (76), and (83):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 4	3	468	24	9	68	12678	5	178
row2	2	589	45689	7	4	1	689	68	3
row3	5679	5789	1	23	235	68	4	2678	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134 4	1347	34	1257	127	6
row6	157	6	25	8	17	9	1257	3	4
row7	4569	2589	7	1249	1245	45 4	3	168	1589
row8	3	589	4589 4	6	145	7	189	18	2
row9	569	1	2569	239	8	35	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX B-----

start

	col 4	col 5	col 6
row1	24	9	68
row2	7	45 <u>4</u>	1
row3	23	235	68

step 1

	col 4	col 5	col 6
	24	9	68
	<u>2</u>		
	7	45 4	1
	23	235	68

step 2

	col 4	col 5	col 6
	24	9	68
	2		
	7	45 4	1
	23	235 35	68
	<u>3</u>		

step 3

	col 4	col 5	col 6
row1	24	9	68
	2		
row2	7	45 4	1
row3	23	235	68
	3	5	

-----BOX E-----

start

	col 4	col 5	col 6
row4	5	6	2
row5	134 <u>4</u>	1347	34
row6	8	17	9

step 1

	col 4	col 5	col 6
	5	6	2
	134	1347	34
	4	137	<u>3</u>
	8	17	9

step 2

	col 4	col 5	col 6
	5	6	2
	134	1347	34
	4	17	3
	8	17	9

--COLUMN 5---		
	start	step1
row1	9	9
	45	45
row2	<u>4</u>	4
	235	235
row3	<u>5</u>	5
row4	6	6
	1347	1347
row5	<u>17</u>	17
	<u>17</u>	17
row6		
	1245	1245
row7		2
	145	145
row8		1
row9	8	8

-----BOX H-----

start

	col 4	col 5	col 6
row7	1249	1245 <u>2</u>	45 <u>4</u>
row8	6	145 <u>1</u>	7
row9	239	8	35

step 1

	col 4	col 5	col 6
row7	1249 <u>9</u>	1245 2	45 4
row8	6	145 1	7
row9	239 39	8	35

step 2

	col 4	col 5	col 6
row7	1249 9	1245 2	45 4
row8	6	145 1	7
row9	239 <u>3</u>	8	35

step 3

	col 4	col 5	col 6
row7	1249 9	1245 2	45 4
row8	6	145 1	7
row9	239 3	8	35 5

But now there are duplicate 3's in squares (34) and (94) of column 4. Subpattern-2 is therefore invalid, making subpattern-3 the winning subpattern. We still need to update the sudoku with subpattern-3 in order to get the solution.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 4	3	468	24 2	9	68	12678	5	178
row2	2	589	45689	7	45 4	1	689	68	3
row3	5679	5789	1	23 3	235 5	68	4	2678	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134 4	1347 17	34 3	1257	127	6
row6	157	6	25	8	17	9	1257	3	4
row7	4569	2589	7	1249 9	1245 2	45 4	3	168	1589
row8	3	589	4589 4	6	145 1	7	189	18	2
row9	569	1	2569	239 3	8	35 5	679	4	579

Subpattern-3 is composed of 4's in squares (14), (23), (56), (71), and (85):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467	3	468	24	9	68	12678	5	178
row2	2	589	45689	7	45	1	689	68	3
row3	5679	5789	1	23	235	68	4	2678	789
row4	17	4	3	5	6	2	178	9	178
row5	8	2579	259	134	1347	34	1257	127	6
row6	157	6	25	8	17	9	1257	3	4
row7	4569	2589	7	1249	1245	45	3	168	1589
row8	3	589	4589	6	145	7	189	18	2
row9	569	1	2569	239	8	35	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	467	3	468	24 <u>4</u>	9	68	12678	5	178
STEP 1	467 67	3	468 <u>68</u>	24 4	9	<u>68</u>	12678	5	178
STEP 2	467 <u>7</u>	3	468 68	24 4	9	68	12678 127	5	178 17
STEP 3	467 7	3	468 68	24 4	9	68	12678 12	5	178 <u>1</u>
STEP 4	467 7	3	468 68	24 4	9	68	12678 2	5	178 1

-----**COLUMN 1**-----

	start	step1	step2	step3
row1	467 <u>7</u>	467 7	467 7	467 7
row2	2	2	2	2
row3	5679	5679 569	5679 569	5679 69
row4	17	17 <u>1</u>	17 1	17 1
row5	8	8	8	8
row6	157	157 15	157 <u>5</u>	157 5
row7	4569 4	4569 4	4569 4	4569 4
row8	3	3	3	3
row9	569	569	569	569 69

-----BOX B-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row1	24 <u>4</u>	9	68	24 4	9	68	24 4	9	68
row2	7	45	1	7	<u>5</u>	1	7	5	1
row3	23	235	68	23	235	68	23	235 23	68

-----BOX H-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row7	1249	1245	45	1249 129	1245 125	45 <u>5</u>	1249 129	1245 12	45 5
row8	6	145 <u>4</u>	7	6	145 4	7	6	145 4	7
row9	239	8	35	239	8	35	239	8	35 <u>3</u>

	step 3		
	col 4	col 5	col 6
row7	1249 129	1245 12	45 5
row8	6	145 4	7
row9	239 29	8	35 3

After updating of row 1, col 1, box B, and box H:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 7	3	468 68	24 4	9	68	12678 2	5	178 1
row2	2	589	45689 4	7	45 5	1	689	68	3
row3	5679 69	5789	1	23	235 23	68	4	2678	789
row4	17 1	4	3	5	6	2	178	9	178
row5	8	2579	259	134	1347	34 4	1257	127	6
row6	157 5	6	25	8	17	9	1257	3	4
row7	4569 4	2589	7	1249 129	1245 12	45 5	3	168	1589
row8	3	589	4589	6	145 4	7	189	18	2
row9	569 69	1	2569	239 29	8	35 3	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4 START	17 <u>1</u>	4	3	5	6	2	178	9	178

STEP 1	17 1	4	3	5	6	2	178 78	9	178 78
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	157 <u>5</u>	6	25	8	17	9	1257	3	4

STEP 1	157 5	6	25 <u>2</u>	8	17	9	1257 127	3	4
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STEP 2	157 5	6	25 2	8	17	9	1257 17	3	4
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-----BOX F-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row4	178 <u>78</u>	9	178 <u>78</u>	178 78	9	178 78	178 78	9	178 78
row5	1257	127	6	1257 125	127 12	6	1257 25	127 <u>2</u>	6
row6	1257 17	3	4	1257 <u>1</u>	3	4	1257 1	3	4

	step 3		
	col 7	col 8	col 9
row4	178 78	9	178 78
row5	1257 <u>5</u>	127 2	6
row6	1257 1	3	4

After updating of row 4, row 6 and box F:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 7	3	468 68	24 4	9	68	12678 2	5	178 1
row2	2	589	45689 4	7	45 5	1	689 68	3	
row3	5679 69	5789	1	23	235 23	68	4	2678	789
row4	17 1	4	3	5	6	2	178 78	9	178 78
row5	8	2579	259	134	1347	34 4	1257 5	127 2	6
row6	157 5	6	25 2	8	17	9	1257 1	3	4
row7	4569 4	2589	7	1249 129	1245 12	45 5	3	168	1589
row8	3	589	4589	6	145 4	7	189	18	2
row9	569 69	1	2569	239 29	8	35 3	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	2	589	45689	7	45 <u>5</u>	1	689	68	3

STEP 1	2	589 89	45689	7	45 5	1	689	68	3
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-----start-----BOX A-----step 1-----

	col 1	col 2	col 3		col 1	col 2	col 3
row1	467 <u>7</u>	3	468 <u>68</u>		467 7	3	468 68
row2	2	589 <u>89</u>	45689 4		2	589 89	45689 4
row3	5679 <u>69</u>	5789	1		5679 69	5789 5	1

COLUMN 2

	start	step1	step2	step3
row1	3	3	3	3
row2	589 89	589 <u>89</u>	589 89	589 89
row3	5789 <u>5</u>	5789 5	5789 5	5789 5
row4	4	4	4	4
row5	2579	2579 279	2579 27	2579 7
row6	6	6	6	6
row7	2589	2589 289	2589 <u>2</u>	2589 2
row8	589	589 <u>89</u>	589 89	589 89
row9	1	1	1	1

After updates of row 2, box A, and col 2:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 7	3	468 68	24 4	9	68	12678 2	5	178 1
row2	2	589 89	45689 4	7	45 5	1	689 68	68	3
row3	5679 69	5789 5	1	23	235 23	68	4	2678	789
row4	17 1	4	3	5	6	2	178 78	9	178 78
row5	8	2579 7	259	134	1347	34 <u>4</u>	1257 5	127 2	6
row6	157 5	6	25 2	8	17	9	1257 1	3	4
row7	4569 <u>4</u>	2589 2	7	1249 129	1245 12	45 5	3	168	1589
row8	3	589 89	4589	6	145 4	7	189	18	2
row9	569 <u>69</u>	1	2569	239 29	8	35 3	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX D-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row4	17 1		3	17 1		3
row5		2579	259		2579	259
	8	7		8	7	9
row6	157		25	157		25
	<u>5</u>	6	<u>2</u>	5	6	2

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	8	2579 <u>7</u>	259 9	134	1347	34 <u>4</u>	1257 5	127 2	6

STEP 1	8	2579 7	259 9	134 13	1347 13	34 4	1257 5	127 2	6
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COLUMN 3

	start	step1
row1	468 68	468 68
row2	45689 <u>4</u>	45689 4
row3	1	1
row4	3	3
row5	259 <u>9</u>	259 9
row6	25 <u>2</u>	25 2
row7	7	7
row8	4589	4589 58
row9	2569	2569 56

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	4569 4	2589 <u>2</u>	7	1249 129	1245 12	45 <u>5</u>	3	168	1589
STEP 1	4569 4	2589 2	7	1249 19	1245 <u>1</u>	45 5	3	168	1589 189
STEP 2	4569 4	2589 2	7	1249 <u>9</u>	1245 1	45 5	3	168 68	1589 89
STEP 3	4569 4	2589 2	7	1249 9	1245 1	45 5	3	168 68	1589 <u>8</u>
STEP 4	4569 4	2589 2	7	1249 9	1245 1	45 5	3	168 6	1589 8

After updates of box D, row 5, col 3, and row 7:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 7	3	468 68	24 4	9	68	12678 2	5	178 1
row2	2	589 89	45689 4	7	45 5	1	689 68	68	3
row3	5679 69	5789 5	1	23	235 23	68	4	2678	789
row4	17 1	4	3	5	6	2	178 78	9	178 78
row5	8	2579 7	259 9	134 13	1347 13	34 4	1257 5	127 2	6
row6	157 5	6	25 2	8	17	9	1257 1	3	4
row7	4569 4	2589 2	7	1249 9	1245 1	45 5	3	168 6	1589 8
row8	3	589 89	4589 58	6	145 4	7	189 18	18	2
row9	569 69	1	2569 56	239 29	8	35 3	679	4	579

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

COLUMN 9

	start	step1	step2	step3
row1	178 1	178 1	178 1	178 1
row2	3	3	3	3
row3	789 79	789 79	789 <u>9</u>	789 9
row4	178 78	178 <u>7</u>	178 7	178 7
row5	6	6	6	6
row6	4	4	4	4
row7	1589 <u>8</u>	1589 8	1589 8	1589 8
row8	2	2	2	2
row9	579	579	579 59	579 5

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	5679 69	5789 5	1	<u>23</u>	235 <u>23</u>	68	4	2678	789 <u>9</u>
STEP 1	5679 <u>6</u>	5789 5	1	23	235 23	68	4	2678 678	789 9
STEP 2	5679 6	5789 5	1	23	235 23	68 <u>8</u>	4	2678 78	789 9
STEP 3	5679 6	5789 5	1	23	235 23	68 8	4	2678 7	789 9

-----BOX C-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row1	12678 2	5	178	12678 2	5	178
row2	689	68	3	689 68	68	3
row3	4	2678 7	789 <u>9</u>	4	2678 7	789 9

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	2	589 89	45689 4	7	45 5	1	689 <u>68</u>	<u>68</u>	3
STEP 1	2	589 9	45689 4	7	45 5	1	689 68	68	3

-----BOX G-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row7	4569 <u>4</u>	2589 <u>2</u>	7	4569 4	2589 2	7
row8	3	589 89	4589	3	589 89	4589 589
row9	569 69	1	2569	569 69	1	2569 569

After updates of col 9, row 3, box C, row 2, and box G:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 7	3	468 68	24 4	9	68	12678 2	5	178 1
row2	2	589 9	45689 4	7	45 5	1	689 68	68	3
row3	5679 6	5789 5	1	23	235 23	68 8	4	2678 7	789 9
row4	17 1	4	3	5	6	2	178 78	9	178 7
row5	8	2579 7	259 9	134 13	1347 13	34 4	1257 5	127 2	6
row6	157 5	6	25 2	8	17	9	1257 1	3	4
row7	4569 4	2589 2	7	1249 9	1245 1	45 5	3	168 6	1589 8
row8	3	589 89	4589 589	6	145 4	7	189	18	2
row9	569 69	1	2569 569	239 29	8	35 3	679	4	579 5

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	569 69	1	2569 569	239 29	8	35 3	679	4	579 <u>5</u>
STEP 1	569 <u>69</u>	1	2569 <u>69</u>	239 29	8	35 3	679	4	579 5
STEP 2	569 69	1	2569 69	239 2	8	35 3	679 7	4	579 5

-----BOX G-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	4569 4	2589 2	4589 7	4569 4	2589 2	4589 7	4569 4	2589 2	4589 7
row2	3	589 89	4589 589	3	589 <u>8</u>	4589 58	3	589 8	4589 5
row3	569 <u>69</u>	1	2569 <u>69</u>	569 69	1	2569 69	569 69	1	2569 69

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	3	589 <u>8</u>	4589 5	6	145 4	7	189	18	2
STEP 1	3	589 8	4589 5	6	145 4	7	189 19	18 <u>1</u>	2
STEP 2	3	589 8	4589 5	6	145 4	7	189 19	18 <u>1</u>	2
STEP 3	3	589 8	4589 5	6	145 4	7	189 9	18 1	2

	COLUMN 1-----		COLUMN 6-----		COLUMN 8-----	
	start	step1	start	step1	start	step1
row1	467 7	467 7	68	68 6	5	5
row2	2	2	1	1	68	68 8
row3	5679 <u>6</u>	5679 6	68 <u>8</u>	68 8	2678 7	2678 7
row4	17 1	17 1	2	2	9	9
row5	8	8	34 4	34 4	127 2	127 2
row6	157 5	157 5	9	9	3	3
row7	4569 4	4569 4	45 5	45 5	168 <u>6</u>	168 6
row8	3	3	7	7	18 1	18 1
row9	569 69	569 9	35 3	35 3	4	4

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	467 7	3	468 68	24 4	9	68 <u>6</u>	12678 2	5	178 1

STEP 1	467 7	3	468 8	24 4	9	68 6	12678 2	5	178 1
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4 START	17 1	4	3	5	6	2	178 78	9	178 <u>7</u>

STEP 1	17 1	4	3	5	6	2	178 8	9	178 7
--------	---------	---	---	---	---	---	-----------------	---	----------

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	2	589 9	45689 4	7	45 5	1	689 68	68 <u>8</u>	3
STEP 1	2	589 9	45689 4	7	45 5	1	689 6	68 8	3
	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	157 5	6	25 2	8	17	9	1257 <u>1</u>	3	4
STEP 1	157 5	6	25 2	8	17 7	9	1257 1	3	4
	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	569 <u>9</u>	1	2569 69	239 2	8	35 3	679 7	4	579 5
STEP 1	569 9	1	2569 6	239 2	8	35 3	679 7	4	579 5

COLUMN 4-----			COLUMN 5-----			
	start	step1	step2	start	step1	step2
row1	24 4	24 4	24 4	9	9	9
row2	7	7	7	45 5	45 5	45 5
row3	23	23 <u>3</u>	23 3	235 23	235 23	235 2
row4	5	5	5	6	6	6
row5	134 13	134 13	134 1	1347 13	1347 <u>3</u>	1347 3
row6	8	8	8	17 7	17 7	17 7
row7	1249 9	1249 9	1249 9	1245 <u>1</u>	1245 1	1245 1
row8	6	6	6	145 4	145 4	145 4
row9	239 <u>2</u>	239 2	239 2	8	8	8

After further updates of row 9, box G, row 8, col 1, col 6, col 8, row 1, row 4, row 2, row 6, row 9 (again), col 4 and col 5:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	467 7	3	468 8	24 4	9	68 6	12678 2	5	178 1
row2	2	589 9	45689 4	7	45 5	1	689 6	68 8	3
row3	5679 6	5789 5	1	23 3	235 2	68 8	4	2678 7	789 9
row4	17 1	4	3	5	6	2	178 8	9	178 7
row5	8	2579 7	259 9	134 1	1347 3	34 4	1257 5	127 2	6
row6	157 5	6	25 2	8	17 7	9	1257 1	3	4
row7	4569 4	2589 2	7	1249 9	1245 1	45 5	3	168 6	1589 8
row8	3	589 8	4589 5	6	145 4	7	189 9	18 1	2
row9	569 9	1	2569 6	239 2	8	35 3	679 7	4	579 5

All rows, columns, and boxes are 9-perfect, so this is the solution.

There are actually two patterns, a larger one in boxes A, B, D and G; and a smaller one boxes E and F. The smaller one seemed too small to be effective, so I opted for the larger pattern. One way to simplify its pattern calculation was to note that the parts of the pattern in boxes B and H were clearly derivative, so I could restrict my attention to just three boxes, A, D and G. Making A the starting box, I could look ahead and see that G would be a pending box for the first two subpatterns, which would begin in square (11). I decided therefore to make the box order **A→G→D for subpatterns 1 and 2, and A→D→G for the remainder.**

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	8	8			8				
row2								X	
row3		8	8		8				
row4		8	8						
row5						8	8		
row6						8	8		
row7									X
row8	8		8	8					
row9			8	8					

We begin subpattern-1 in square (11). The box order is $A \rightarrow G \rightarrow D$, so box G is next, and we have two choices. We choose square (83), mark square (93) as the pending square, and write a pending P in the box. In the final box, D, the choice of square (42) is forced. Back at the starting box, a pending condition exists, so subpattern-2 is begun in the same square as subpattern-1. The next box G is pending, so subpattern-2 is continued in the pending square, (93), and the pending P is erased. In the final box D

	col 1	col 2	col 3
row1	8 ₁₂	8	
row2			
row3		8	8
row4		8 ₁₂	8
row5			
row6			
row7			<i>P</i>
row8	8		8 ₁
row9			8 ₂

Back at the starting box, there is no longer a pending condition, so we may change the order of boxes to $A \rightarrow D \rightarrow G$. We begin subpattern-3 in square (12). In boxes D and G the choices are forced, as they are in the remainder of subpatterns. The remainder of the three-box part of the analysis is simple, and will be left to the reader. On the next page we shall compute the rest of the pattern.

	col 1	col 2	col 3
row1	8_{12}	8_3	
row2			
row3		8_4	8_5
row4		8_{125}	8_{34}
row5			
row6			
row7			$\neq$
row8	8_{345}		8_1
row9			8_2

To complete the pattern, we use note that box B is a derivative of box A, and that box H is a derivative of box G, and use the law of total subscripts to calculate the subscripts for the two derivative boxes.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	8 ₁₂	8 ₃			8 ₄₅				
row2								X	
row3		8 ₄	8 ₅		8 ₁₂₃				
row4		8 ₁₂₅	8 ₃₄						
row5						8 _A	8 _B		
row6						8 _B	8 _A		
row7									X
row8	8 ₃₄₅		8 ₁	8 ₂					
row9			8 ₂	8 ₁₃₄₅					

The testing was grueling. Subpatterns 1, 2 and 3 all failed, subpattern 4 jammed up, and subpattern-5 worked. Whew! After going to all this trouble, I decided to look at the far simpler sister pattern in boxes E and F. Its pattern is too simple to explain, and is noted above. It's hard to believe that such a simple pattern might work, but it doesn't hurt to try.

We'll first try subpattern-1, with 8's in squares (56) and (67):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	45678	678	9	3	58	14	456	157	2
row2	4567	1	356	26	9	24	3456	8	3567
row3	2	368	34568	16	58	7	9	145	1356
row4	9	368	1368	4	26	5	7	12	13
row5	157	4	15	127	3	128	258	6	9
row6	1567	367	2	9	67	18	358	15	4
row7	46	2	7	5	1	9	46	3	8
row8	168	5	168	278	4	3	26	9	67
row9	3	9	48	78	27	6	1	2457	57

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	1567	367	2	9	67	18	358	15	4
							<u>8</u>		
STEP 1	1567	367	2	9	67	18	358	15	4
						<u>1</u>	8		
STEP 2	1567	367	2	9	67	18	358	15	4
	567					1	8	<u>5</u>	
STEP 3	1567	367	2	9	67	18	358	15	4
	<u>67</u>					1	8	5	
STEP 4	1567	367	2	9	67	18	358	15	4
	67	3				1	8	5	

-----BOX F-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 1	col 2	col 3
row4	7	12	13	7	12	13	7	<u>1</u>	13
row5	258	6	9	258	6	9	258	6	9
row6	358	15	4	358	15	4	358	15	4
	<u>8</u>	<u>5</u>		8	5		8	5	
step 3									
	col 7	col 8	col 9						
row4	7	12	13						
		1	3						
row5	258	6	9						
	2								
row6	358	15	4						
	8	5							

	COLUMN 6-----		
	start	step1	step2
row1	14	14	14
		<u>4</u>	4
row2	24	24	24
			2
row3	7	7	7
row4	5	5	5
row5	128	128	128
	8	8	8
	18	18	18
row6	<u>1</u>	1	1
row7	9	9	9
row8	3	3	3
row9	6	6	6

COLUMN 7-----					
	start	step1	step2	step3	step4
row1	456	456	456 45	456 <u>5</u>	456 5
row2	3456	3456	3456 345	3456 35	3456 3
row3	9	9	9	9	9
row4	7	7	7	7	7
row5	258 <u>2</u>	258 2	258 2	258 2	258 2
row6	358 8	358 8	358 8	358 8	358 8
row7	46	46	46 <u>4</u>	46 4	46 4
row8	26	26 <u>6</u>	26 6	26 6	26 6
row9	1	1	1	1	1

COLUMN 8-----			
	start	step1	step2
row1	157	157 <u>7</u>	157 7
row2	8	8	8
row3	145	145	145
row4	12 <u>1</u>	12 1	12 1
row5	6	6	6
row6	15 <u>5</u>	15 5	15 5
row7	3	3	3
row8	9	9	9
row9	2457	2457 247	2457 2

-----BOX I-----									
	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row7	46 4	3	8	46 4	3	8	46 4	3	8
row8	26 <u>6</u>	9	67	26 6	9	<u>7</u>	26 6	9	7
row9	1	2457 2	57	1	2457 2	57	1	2457 2	57 5

After updates of row 6, box F, col 6, col 7, col 8 box I:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	45678	678	9	3	58	14	456	157	
row2	4567	1	356	26	9	24	3456	8	3567
row3	2	368	34568	16	58	7	9	145	1356
row4	9	368	1368	4	26	5	7	12	13
row5	157	4	15	127	3	128	258	6	9
row6	1567 67	3	2	9	67	18 1	358 8	15 5	4
row7	46	2	7	5	1	9	46 4	3	8
row8	168	5	168	278	4	3	26 6	9	67 7
row9	3	9	48	78	27	6	1	2457 2	57 5

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX C-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row1	456 <u>5</u>	157 <u>7</u>	2	456 5	157 7	2	456 5	157 7	2
row2	3456 <u>3</u>	8	3567	3456 3	8	<u>6</u>	3456 3	8	3567 6
row3	9	145 4	1356	9	145 4	1356 16	9	145 4	1356 1

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	4567	1	356	26	9	24	3456 <u>3</u>	8	3567 <u>6</u>
STEP 1	4567 457	1	356 <u>5</u>	26 2	9	24 2	3456 3	8	3567 6
STEP 2	4567 47	1	356 5	26 <u>2</u>	9	24 <u>2</u>	3456 3	8	3567 6

But now there are two 2's in row 2, so subpattern-1 is invalid.

This leaves subpattern-2 as the correct subpattern, and we must use it to determine the solution. Subpattern-2 has 8's in squares (57) and (66).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	45678	678	9	3	58	14	456	157	2
row2	4567	1	356	26	9	24	3456	8	3567
row3	2	368	34568	16	58	7	9	145	1356
row4	9	368	1368	4	26	5	7	12	13
row5	157	4	15	127	3	128	258	6	9
row6	1567	367	2	9	67	18	358	15	4
row7	46	2	7	5	1	9	46	3	8
row8	168	5	168	278	4	3	26	9	67
row9	3	9	48	78	27	6	1	2457	57

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX F-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row4	7	12	13	7	12	<u>13</u>	7	2	13
row5	258			258			258		
	<u>8</u>	6	9	8	6	9	8	6	9
row6	358	15		358	<u>15</u>		358	15	
			4	<u>35</u>		4	35		4

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4 START	9	368	1368	4	26	5	7	12	13
								<u>2</u>	
STEP 1	9	368	1368	4	<u>6</u>	5	7	2	13
STEP 2	9	38	138	4	6	5	7	2	13

-----BOX E-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row4	4	<u>6</u>	5	4	6	5	4	6	5
row5	127		128	127		128	127		128
		3			3	12	12	3	12
row6	9	67	18	9	67	18	9	7	18
			<u>8</u>		<u>7</u>	8			8

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	157	4	15	127 <u>12</u>	3	128 <u>12</u>	258 8	6	9
STEP 1	157 57	4	15 <u>5</u>	127 12	3	128 12	258 8	6	9
STEP 2	157 7	4	15 5	127 12	3	128 12	258 8	6	9

-----BOX D-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row4	9	368 38	1368 138	9	368 38	1368 138
row5	157 <u>7</u>	4	15 <u>5</u>	157 7	4	15 5
row6	1567	367	2	1567 16	367 36	2

-----BOX A-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row1	45678	678 <u>7</u>	9	45678 4568	678 7	9
row2	4567	1	356	4567 456	1	356
row3	2	368	34568	2	368	34568

After updates of box F, row 4, box E, row 5, box D, and box A:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	45678 4568	678 7	9	3	58	14	456	157	2
row2	4567 456	1	356	26	9	24	3456	8	3567
row3	2	368	34568	16	58	7	9	145	1356
row4	9	368 38	1368 138	4	26 6	5	7	12 2	13
row5	157 7	4	15 5	127 12	3	128 12	258 8	6	9
row6	1567 16	367 36	2	9	67 7	18 8	358 35	15	4
row7	46	2	7	5	1	9	46	3	8
row8	168	5	168	278	4	3	26	9	67
row9	3	9	48	78	27	6	1	2457	57

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1	45678 4568	678 <u>7</u>	9	3	58	14	456	157	2
STEP 1	45678 4568	7	9	3	58	14	456	157 15	2

-----BOX C-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row1	<u>456</u>	157 <u>15</u>	2	456	157 15	2
row2	<u>3456</u>	8	3567	3456	8	3567 <u>7</u>
row3	9	<u>145</u>	<u>1356</u>	9	145	1356

	COLUMN 9-----			COLUMN 8-----		
	start	step1	step2	start	step1	step2
row1	2	2	2	157 <u>15</u>	157 15	157 15
row2	3567 <u>7</u>	3567 7	3567 7	8	8	8
row3	1356	1356	1356 13	145	145 <u>4</u>	145 4
row4	13	13	13	12 <u>2</u>	12 2	12 2
row5	9	9	9	6	6	6
row6	4	4	4	<u>15</u>	15	15
row7	8	8	8	3	3	3
row8	67	67 <u>6</u>	67 6	9	9	9
row9	57	57 <u>5</u>	57 5	2457	2457 47	2457 7

-----BOX I-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row7	46	3	8	46	3	8
				4		
row8	26	9	67	26	9	67
				2		6
row9	1	2457	57	1	2457	57
		7	5		7	5

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	168	5	168	278	4	3	26 <u>2</u>	9	67 <u>6</u>
STEP 1	168 <u>18</u>	5	168 <u>18</u>	278 78	4	3	26 2	9	67 6
STEP 2	168 18	5	168 18	278 7	4	3	26 2	9	67 6
ROW 9 START	3	9	48	78	27	6	1	2457 <u>7</u>	57 5
STEP 1	3	9	48	78 <u>8</u>	27 2	6	1	2457 7	57 5
STEP 2	3	9	48 4	78 8	27 2	6	1	2457 7	57 5

After ROW 1, BOX C, COL 9, COL 8, BOX I, ROW 8, and ROW 9:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	45678 4568	678 7	9	3	58	14	456	157 15	2
row2	4567 456	1	356	26	9	24	3456	8	3567 7
row3	2	368	34568	16	58	7	9	145 4	1356 13
row4	9	368 38	1368 138	4	26 6	5	7	12 2	13
row5	157 7	4	15 5	127 12	3	128 12	258 8	6	9
row6	1567 16	367 36	2	9	67 7	18 8	358 35	15	4
row7	46	2	7	5	1	9	46 4	3	8
row8	168 18	5	168 18	278 7	4	3	26 2	9	67 6
row9	3	9	48 4	78 8	27 2	6	1	2457 7	57 5

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

COLUMN 3	
	start
row1	9
	356
row2	
	34568
row3	
	1368
row4	138
	15
row5	<u>5</u>
row6	2
row7	7
	168
row8	18
	48
row9	<u>4</u>

step1

9
356
36
34568
368
1368
138
15
5
2
7
168
18
48
4

BOX A

	col 1	col 2	col 3
row1	45678 4568	678 7	9
row2	4567 456	1	356 36
row3	2	368	34568 368

	step1		
row1	45678 45	678 7	9
row2	4567 45	1	356 36
row3	2	368	34568 368

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	45678 45	678 7	9	3	58	14	456	157 15	2
STEP 1	45678 45	678 7	9	3	58 8	14	456 6	157 15	2

-----start-----BOX B-----step 1-----

	col 4	col 5	col 6
row1	3	58 8	14
row2	26	9	24
row3	16	58	7

	col 4	col 5	col 6
row1	3	58 8	14
row2	26	9	24
row3	16	58 5	7

-----BOX C-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row1	456 <u>6</u>	157 15	2	456 6	157 15	2
row2	3456	8	3567	3456 35	8	3567 7
row3	9	145 <u>4</u>	1356 13	9	145 4	1356 13

COLUMN 1-----

	start	step1	step2	step3
row1	45678 <u>45</u>	45678 45	45678 45	45678 45
row2	4567 <u>45</u>	4567 45	4567 45	4567 45
row3	2	2	2	2
row4	9	9	9	9
row5	157 7	157 7	157 7	157 7
row6	1567 16	1567 16	1567 <u>1</u>	1567 1
row7	46	46 <u>6</u>	46 6	46 6
row8	168 18	168 18	168 18	168 8
row9	3	3	3	3

After updates of COL 3, BOX A, ROW 1, BOX B, BOX C, and COL 1:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	45678 45	678 7	9	3	58 8	14	456 6	157 15	2
row2	4567 45	1	356 36	26	9	24	3456 35	8	3567 7
row3	2	368	34568 368	16	58 5	7	9	145 4	1356 13
row4	9	368 38	1368 138	4	26 6	5	7	12 2	13
row5	157 7	4	15 5	127 12	3	128 12	258 8	6	9
row6	1567 1	367 36	2	9	67 7	18 8	358 35	15	4
row7	46 6	2	7	5	1	9	46 4	3	8
row8	168 8	5	168 18	278 7	4	3	26 2	9	67 6
row9	3	9	48 4	78 8	27 2	6	1	2457 7	57 5

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	1567 <u>1</u>	367 36	2	9	67 7	18 8	358 35	15	4
STEP 1	1567 1	367 36	2	9	67 7	18 8	358 35	15 <u>5</u>	4
STEP 2	1567 1	367 36	2	9	67 7	18 8	358 <u>3</u>	15 5	4
STEP 3	1567 1	367 6	2	9	67 7	18 8	358 3	15 5	4

-----BOX F-----

	start			step 1		
	col 7	col 8	col 9	col 7	col 8	col 9
row4	7	12 2	13	7	12 2	13 1
row5	258 8	6	9	258 8	6	9
row6	358 <u>3</u>	15 5	4	358 3	15 5	4

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	168 <u>8</u>	5	168 18	278 7	4	3	26 2	9	67 6
STEP 1	168 8	5	168 1	278 7	4	3	26 2	9	67 6

	COLUMN 2-----		COLUMN 3-----		COLUMN 7-----	
	start	step1	start	step1	start	step1
row1	678 7	678 7	9	9	456 6	456 6
row2	1	1	356 36	356 36	3456 35	3456 5
row3	368	368 38	34568 368	34568 368	9	9
row4	368 38	368 38	1368 138	1368 38	7	7
row5	4	4	15 5	15 5	258 8	258 8
row6	367 <u>6</u>	367 6	2	2	358 <u>3</u>	358 3
row7	2	2	7	7	46 4	46 4
row8	5	5	168 <u>1</u>	168 1	26 2	26 2
row9	9	9	48 4	48 4	1	1

-----BOX C-----

	start			step 1			step 2		
	col 7	col 8	col 9	col 7	col 8	col 9	col 7	col 8	col 9
row1	456 6	157 15	2	456 6	157 <u>1</u>	2	456 6	157 1	2
row2	3456 <u>5</u>	8	3567 7	3456 5	8	3567 7	3456 5	8	3567 7
row3	9	145 4	1356 13	9	145 4	1356 13	9	145 4	1356 3

After updates of ROW 6, BOX F, ROW 8, COL 2, COL 3, COL 7, and BOX C:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	45678 45	678 7	9	3	58 8	14	456 6	157 1	2
row2	4567 45	1	356 36	26	9	24	3456 5	8	3567 7
row3	2	368 38	34568 368	16	58 5	7	9	145 4	1356 3
row4	9	368 38	1368 38	4	26 6	5	7	12 2	13 1
row5	157 7	4	15 5	127 12	3	128 12	258 8	6	9
row6	1567 1	367 6	2	9	67 7	18 8	358 3	15 5	4
row7	46 6	2	7	5	1	9	46 4	3	8
row8	168 8	5	168 1	278 7	4	3	26 2	9	67 6
row9	3	9	48 4	78 8	27 2	6	1	2457 7	57 5

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	45678 45	678 7	9	3	58 8	14	456 6	157 1	2
STEP 1	45678 45	678 7	9	3	58 8	14 <u>4</u>	456 6	157 1	2
STEP 2	45678 5	678 7	9	3	58 8	14 4	456 6	157 1	2

-----BOX B-----

	start			step 1			step 2		
	col 4	col 5	col 6	col 4	col 5	col 6	col 4	col 5	col 6
row1	3	58 8	14 <u>4</u>	3	58 8	14 4	3	58 8	14 4
row2	26	9	24	26	9	24 <u>2</u>	26 <u>6</u>	9	24 2
row3	16	58 5	7	16	58 5	7	16	58 5	7
	step 3								
	col 4	col 5	col 6						
row1	3	58 8	14 4						
row2	26 6	9	24 2						
row3	16 1	58 5	7						

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	4567 45	1	356 36	26 <u>6</u>	9	24 2	3456 <u>5</u>	8	3567 7
STEP 1	4567 4	1	356 3	26 6	9	24 2	3456 5	8	3567 7

-----BOX A-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	45678 5	678 7		45678 5	678 7		45678 5	678 7	
row2	4567 4	1	356 <u>3</u>	4567 4	1	356 3	4567 4	1	356 3
row3		368 38	34568 368		368 <u>8</u>	34568 68		368 8	34568 6

	COLUMN 2-----		COLUMN 3-----		COLUMN 4-----	
	start	step1	step2	step3	step4	step5
row1	678 7	678 7	9	9	3	3
row2	1	1	356 <u>3</u>	356 3	26 6	26 6
row3	368 <u>8</u>	368 8	34568 6	34568 6	16 <u>1</u>	16 1
row4	368 38	368 3	1368 38	1368 8	4	4
row5	4	4	15 5	15 5	127 12	127 2
row6	367 6	367 6	2	2	9	9
row7	2	2	7	7	5	5
row8	5	5	168 1	168 1	278 7	278 7
row9	9	9	48 4	48 4	78 8	78 8

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	157 7	4	15 5	127 <u>2</u>	3	128 12	258 8	6	9
STEP 1	157 7	4	15 5	127 2	3	128 1	258 8	6	9
	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4 START	9	368 38	1368 <u>8</u>	4	26 6	5	7	12 2	13 1
STEP 1	9	368 3	1368 8	4	26 6	5	7	12 2	13 1

After updates of ROW 1, BOX B, ROW 2, BOX A, COL 2, COL 3, COL 4, ROW 5, and ROW 4:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	45678 5	678 7	35678 9	26 3	58 8	14 4	456 6	157 1	3567 2
row2	4567 4	1 1	356 3	26 6	9 9	24 2	3456 5	8 8	3567 7
row3	2 2	368 8	34568 6	16 1	58 5	7 7	9 9	145 4	1356 3
row4	9 9	368 3	1368 8	4 4	26 6	5 5	7 7	12 2	13 1
row5	157 7	4 4	15 5	127 2	3 3	128 1	258 8	6 6	9 9
row6	1567 1	367 6	2 2	9 9	67 7	18 8	358 3	15 5	4 4
row7	46 6	2 2	7 7	5 5	1 1	9 9	46 4	3 3	8 8
row8	168 8	5 5	168 1	278 7	4 4	3 3	26 2	9 9	67 6
row9	3 3	9 9	48 4	78 8	27 2	6 6	1 1	2457 7	57 5

A check of all rows, columns and boxes shows them all to be 9-perfect. This is the solution.

Example N. A Partial Pattern Solution

The sudoku below has been taken as far as standard solving techniques can manage, but the number of digits for every digit pattern are either too few or too many. The patterns for digits 1, 2, 3 and 5 are too simple to reduce candidates sufficiently for normal techniques to deal with. In desperation, the 4-pattern was re-investigated.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	168	3	9	678	678	1678	5	4
row2	46	5	9	24678	1	4678	678	3	278
row3	7	468	148	2468	3	5	9	68	128
row4	8	9	2	467	67	1	3	467	5
row5	45	3	45	678	2	9	678	1	78
row6	1	7	6	3	5	48	248	248	9
row7	34	148	7	5	9	38	1248	248	6
row8	356	2	158	1678	4	3678	78	9	178
row9	9	468	148	1678	678	2	5	478	3

	1		2			3	3	3			4	5	5	5
1	1	1	2	2		3	3	3					5	5
			2	2				3		4			5	5

			7						9	9	9
6			7			8			9	9	9
		6	7						9	9	9

A	B	
D	E	F
G		I

&

↓	←	
•	↑	←
→		↑

This is the 4-pattern. Since box G has the most candidates, it is chosen for the starting box. The initial box order is chosen to be $G \rightarrow I \rightarrow F \rightarrow E \rightarrow B \rightarrow A \rightarrow D$, with square (17) as the beginning square of subpattern-1. With this box order, the choices for subpattern-1 are all forced (see below). With square (72) chosen for the beginning square of subpattern-2, the choices for its extensions are also all forced, and occupy the same squares as subpattern-1 for the first four extensions.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1									
row2	4			4		4 ₁₂			
row3		4 ₁	4 ₂	4					
row4				4 ₁₂				4	
row5	4 ₂		4 ₁						
row6						4	4 ₁₂	4	
row7	4 ₁	4 ₂					4	4	
row8									
row9		4	4					4 ₁₂	

When we go to subpattern-3, the future starts to look a little grim, with **three pending boxes** I, F and B (note the use of the subscript 0 for the pending squares in the pending boxes). Each pending box adds an additional subscript, so subpatterns 4, 5 and 6 will all be additional subscripts created by the pending boxes. Subpattern-7 will have to start in square (93), and will probably encounter an equal number of pending boxes. We might end up with as many as nine or ten subpatterns. We might be lucky and find subpattern-1 solves the sudoku, but maybe subpattern-10 will turn out to be the winner. What to do?

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1					P_3				
row2	4			4_3		4_{12}			
row3		4_1	4_{23}	4_0					
row4				4_{12}				4_3	
row5	4_{23}		4_1						P_2
row6						4_3	4_{12}	4_0	
row7	4_1	4_2					4_3	4_0	
row8									P_1
row9		4_3	4_{7-9}					4_{12}	

The first thought that occurs to me is to give up the analysis for the moment. What can I do with the little information I've gathered so far? I can see that subpatterns-1 and -2 each has a square to itself in box G, and that will not change. No more subpatterns can begin in squares (71) or (72). Suppose I test out the first two subpatterns? If one of them solves the sudoku, I'm done. If not, I can remove the 4-candidates from squares (71) and (72), in both the pattern above, and in the sudoku itself, and this may simplify the sudoku sufficiently to make it more readily soluble. I can also cancel the 4-candidate from square (98). One can see by the geometry of the situation that only subpatterns-1 and -2 can occupy square (98), so the 4-candidate may be removed from that square as well.

We'll first test 4-subpattern-1, which we can see by the analysis above occupies squares (26), (32), (44), (53), (67), (71) and (98).

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	168	3	9	678	678	1678	5	4
row2	46	5	9	24678	1	4678	678	3	278
row3	7	468	148	2468	3	5	9	68	128
row4	8	9	2	467	67	1	3	467	5
row5	45	3	45	678	2	9	678	1	78
row6	1	7	6	3	5	48	248	248	9
row7	34	148	7	5	9	38	1248	248	6
row8	356	2	158	1678	4	3678	78	9	178
row9	9	468	148	1678	678	2	5	478	3

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	1	7	6	3	5	48	248 <u>4</u>	248	9
STEP 1	1	7	6	3	5	48 <u>8</u>	248 4	248 28	9
STEP 2	1	7	6	3	5	48 8	248 4	248 2	9

	COLUMN 8			
	start	step1	step2	step3
row1	5	5	5	5
row2	3	3	3	3
row3	68	68	68 <u>6</u>	68 6
row4	467	467 67	467 67	467 7
row5	1	1	1	1
row6	248 <u>2</u>	248 2	248 2	248 2
row7	248	248 <u>8</u>	248 8	248 8
row8	9	9	9	9
row9	478 <u>4</u>	478 4	478 4	478 4

	COLUMN 3		
	start	step1	step2
row1	3	3	3
row2	9	9	9
row3	148	148	148
		<u>18</u>	18
row4	2	2	2
row5	45	45	45
	<u>4</u>	4	4
row6	6	6	6
row7	7	7	7
row8	158	158	158
			5
row9	148	148	148
		<u>18</u>	18

	COLUMN 1		
	start	step1	step2
row1	2	2	2
	46	46	46
row2		<u>6</u>	6
row3	7	7	7
row4	8	8	8
	45	45	45
row5		<u>5</u>	5
row6	1	1	1
	34	34	34
row7	<u>4</u>	4	4
	356	356	356
row8			3
row9	9	9	9

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7	34	148				38	1248	248	
	<u>4</u>		7	5	9			<u>8</u>	6
STEP 1	34	148				38	1248	248	
	4	<u>1</u>	7	5	9	3	12	8	6
STEP 2	34	148				38	1248	248	
	4	1	7	5	9	3	2	8	6

-----BOX I-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	1248 2	248 <u>8</u>	6	1248 2	248 8	6	1248 2	248 8	6
row2	78	9	178	78 <u>7</u>	9	178 17	78 7	9	178 1
row3	5	478 4	3	5	478 4	3	5	478 4	3

After updating of row 6, col 8, col 3, col 1, row 7, and box I:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	168	3	9	678	678	1678	5	4
row2	46 6	5	9	24678	1	4678 4	678	3	278
row3	7	468 4	148 18	2468	3	5	9	68 6	128
row4	8	9	2	467 4	67	1	3	467 7	5
row5	45 5	3	45 4	678	2	9	678	1	78
row6	1	7	6	3	5	48 8	248 4	248 2	9
row7	34 4	148 1	7	5	9	38 3	1248 2	248 8	6
row8	356 3	2	158 5	1678	4	3678	78 7	9	178 1
row9	9	468	148 18	1678	678	2	5	478 4	3

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX G-----

	start	step 1	step 2
	col 1	col 2	col 3
row1	34 <u>4</u>	148 <u>1</u>	7
row2	356 3	2	158 5
row3	9	468	148 18

	col 1	col 2	col 3
row1	34 4	148 1	7
row2	356 3	2	158 5
row3	9	468 68	148 <u>8</u>

	col 1	col 2	col 3
row1	34 4	148 1	7
row2	356 3	2	158 5
row3	9	468 6	148 8

-----BOX F-----

	start	step 1	step 2
	col 1	col 2	col 3
row1	3	467 <u>7</u>	5
row2	678	1	78
row3	248 4	248 2	9

	col 1	col 2	col 3
row1	3	467 7	5
row2	678 68	1	78 <u>8</u>
row3	248 4	248 2	9

	col 1	col 2	col 3
row1	3	467 7	5
row2	678 6	1	78 8
row3	248 4	248 2	9

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5 START	45 5		45 4	678	2	9	678 <u>6</u>	1	78 <u>8</u>

STEP 1	45 5		45 4	678 7	2	9	678 6	1	78 8
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	COLUMN 9		
	start	step1	step2
row1	4	4	4
	278	278	278
row2		27	7
	128	128	128
row3		<u>2</u>	2
row4	5	5	5
	78	78	78
row5	<u>8</u>	8	8
row6	9	9	9
row7	6	6	6
	178	178	178
row8	<u>1</u>	1	1
row9	3	3	3

-----BOX C-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	1678	5	4	1678 18	5	4	1678 1	5	4
row2	678	3	<u>7</u>	678 <u>8</u>	3	7	678 8	3	7
row3	9	68 <u>6</u>	128	9	68	128	9	68	128

After updates of box G, box F, row 5, col 9 and box C:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	168	3	9	678	678	1678	5	4
row2	46 6	5	9	24678	1	4678 4	678 8	3	278 7
row3	7	468 4	148 18	2468	3	5	9	68 6	128 2
row4	8	9	2	467 4	67	1	3	467 7	5
row5	45 5	3	45 4	678 7	2	9	678 6	1	78 8
row6	1	7	6	3	5	48 8	248 4	248 2	9
row7	34 4	148 1	7	5	9	38 3	1248 2	248 8	6
row8	356 3	2	158 5	1678	4	3678	78 7	9	178 1
row9	9	468 6	148 8	1678	678	2	5	478 4	3

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2 START	46 <u>6</u>	5	9	24678	1	4678 <u>4</u>	678 <u>8</u>	3	278 <u>7</u>

STEP 1	46 6	5	9	24678 2	1	4678 4	678 8	3	278 7
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	7	468 <u>4</u>	148 18	2468	3	5	9	68 <u>6</u>	128 <u>2</u>

STEP 1	7	468 4	148 18	2468 8	3	5	9	68 6	128 2
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STEP 2	7	468 4	148 1	2468 8	3	5	9	68 6	128 2
--------	---	----------	-----------------	-----------	---	---	---	---------	----------

COLUMN 2

	start	step1
row1	168	168 8
row2	5	5
row3	468 4	468 4
row4	9	9
row5	3	3
row6	7	7
row7	148 <u>1</u>	148 1
row8	2	2
row9	468 <u>6</u>	468 6

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	9	468 <u>6</u>	148 <u>8</u>	1678	678	2	5	478 4	3
STEP 1	9	468 6	148 8	1678 17	678 <u>7</u>	2	5	478 4	3
STEP 2	9	468 6	148 8	1678 1	678 7	2	5	478 4	3

	COLUMN 5		
	start	step1	step2
row1	678	678 68	678 8
row2	1	1	1
row3	3	3	3
row4	67	67 <u>6</u>	67 6
row5	2	2	2
row6	5	5	5
row7	9	9	9
row8	4	4	4
row9	678 <u>7</u>	678 7	678 7

After updates of row 2, row 3, col 2, row 9 and col 5:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	8	3	9	8	678	1678	5	4
row2	46 6	5	9	24678 2	1	4678 4	678 8	3	278 7
row3	7	468 4	148 1	2468 8	3	5	9	68 6	128 2
row4	8	9	2	467 4	67 6	1	3	467 7	5
row5	45 5	3	45 4	678 7	2	9	678 6	1	78 8
row6	1	7	6	3	5	48 8	248 4	248 2	9
row7	34 4	148 1	7	5	9	38 3	1248 2	248 8	6
row8	356 3	2	158 5	1678	4	3678	78 7	9	178 1
row9	9	468 6	148 8	1678 1	678 7	2	5	478 4	3

But now there are two 8's in box B, so subpattern-1 fails.

Next, we'll test subpattern-2, with 4-candidates in squares (26), (33), (44), (51), (67), (72), and (98):

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	168	3	9	678	678	1678	5	4
row2	46	5	9	24678	1	4678	678	3	278
row3	7	468	148	2468	3	5	9	68	128
row4	8	9	2	467	67	1	3	467	5
row5	45	3	45	678	2	9	678	1	78
row6	1	7	6	3	5	48	248	248	9
row7	34	148	7	5	9	38	1248	248	6
row8	356	2	158	1678	4	3678	78	9	178
row9	9	468	148	1678	678	2	5	478	3

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	34	148	7	5	9	38	1248	248	6
STEP 1	34	4	7	5	9	38	1248	248	6
STEP 2	34	4	7	5	9	8	1248	248	6
STEP 3	34	4	7	5	9	8	1	2	6

After updating row 7, we immediately see a contradiction. The three squares in column 6 - (26), (66) and (76) - contain just two candidates (4 and 8). Therefore subpattern-2 is invalid.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	168	3	9	678	678	1678	5	4
row2	46	5	9	24678	1	4678	678	3	278
row3	7	468	148	2468	3	5	9	68	128
row4	8	9	2	467	67	1	3	467	5
row5	45	4	3	45	678	2	9	678	1
row6	1	7	6	3	5	48	248	248	9
row7	34	148	7	5	9	38	1248	248	6
row8	356	2	158	1678	4	3678	78	9	178
row9	9	468	148	1678	678	2	5	478	3

Since subpatterns-1 and -2 are both invalid, we may remove these candidates from the squares that contain ONLY one or both of these candidates. Looking at the 4-pattern, we see that there are only three squares which have this characteristic, namely squares (71), (72) and (98).

On the next page, we'll perform the removal of the 4-candidates from these three squares, and see if this leads to a solution.

Note the three changes. The more important are square (71), whose 3-candidate is now the established value, and squares (87) and (98), which contain a pair of 78's.

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	168	3	9	678	678	1678	5	4
row2	46	5	9	24678	1	4678	678	3	278
row3	7	468	148	2468	3	5	9	68	128
row4	8	9	2	467	67	1	3	467	5
row5	45	3	45	678	2	9	678	1	78
row6	1	7	6	3	5	48	248	248	9
row7	3	18	7	5	9	38	1248	248	6
row8	356	2	158	1678	4	3678	78	9	178
row9	9	468	148	1678	678	2	5	78	3

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	168	3	9	678	678	1678	5	4
row2	46	5	9	24678	1	4678	678	3	278
row3	7	468	148	2468	3	5	9	68	128
row4	8	9	2	467	67	1	3	467	5
row5	45	3	45	678	2	9	678	1	78
row6	1	7	6	3	5	48	248	248	9
row7	3	18	7	5	9	38	1248	248	6
row8	56	2	158	1678	4	3678	78	9	178
row9	9	468	148	1678	678	2	5	78	3

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 7 START	<u>3</u>	18	7	5	9	38	1248	248	6
STEP 1	3	18	7	5	9	<u>8</u>	1248	248	6
STEP 2	3	<u>1</u>	7	5	9	8	1248 124	248 24	6
STEP 3	3	1	7	5	9	8	1248 24	248 24	6

	start	step1	step2	step3
row1	678	678 67	678 <u>67</u>	678 67
row2	4678	4678 467	4678 67	4678 67
row3	5	5	5	5
row4	1	1	1	1
row5	9	9	9	9
row6	48	48 <u>4</u>	48 4	48 4
row7	38 <u>8</u>	38 8	38 8	38 8
row8	3678	3678 367	3678 367	3678 3
row9	2	2	2	2

-----BOX B-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	9	678	678 <u>67</u>	9	<u>8</u>	678 67	9	8	678 67
row2	24678	1	4678 <u>67</u>	24678 248	1	4678 67	24678 24	1	4678 67
row3	2468	3	5	2468 248	3	5	2468 24	3	5

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	1	7	6	3	5	48 <u>4</u>	248	248	9
STEP 1	1	7	6	3	5	48 4	248 28	248 28	9

-----BOX F-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	3	467	5	3	467	5	3	46 46	5
row2	678	1	78	678 67	1	<u>7</u>	678 <u>6</u>	1	78 7
row3	248 <u>28</u>	248 <u>28</u>	9	248 28	248 28	9	248 28	248 28	9

	step 3		
	col 1	col 2	col 3
row1	3	4	5
row2	678 6	1	78 7
row3	248 28	248 28	9

COLUMN 8				
	start	step1	step2	step3
row1	5	5	5	5
row2	3	3	3	3
row3	68	68	68	68
				6
row4	467	467	467	467
	<u>4</u>	4	4	4
row5	1	1	1	1
	248	248	248	248
row6	28	28	<u>8</u>	8
	248	248	248	248
row7	24	<u>2</u>	2	2
row8	9	9	9	9
	78	78	78	78
row9				7

After updates of row 7, col 6, box B, row 6, box F, and col 8:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	168	3	9	678 8	678 67	1678	5	4
row2	46	5	9	24678 24	1	4678 67	678	3	278
row3	7	468	148	2468 24	3	5	9	68 6	128
row4	8	9	2	467	67	1	3	467 4	5
row5	45	3	45	678	2	9	678 6	1	78 7
row6	1	7	6	3	5	48 4	248 28	248 8	9
row7	3	18 1	7	5	9	38 8	1248 24	248 2	6
row8	56	2	158	1678	4	3678 3	78	9	178 1
row9	9	468	148	1678	678	2	5	78 7	3

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 6 START	1	7	6	3	5	⁴⁸ 4	²⁴⁸ 28	²⁴⁸ <u>8</u>	9
STEP 1	1	7	6	3	5	⁴⁸ 4	²⁴⁸ 2	²⁴⁸ 8	9

	COLUMN 7		
	start	step1	step2
row1	1678	1678 178	1678 1
row2	678	678 <u>78</u>	678 78
row3	9	9	9
row4	3	3	3
row5	⁶⁷⁸ <u>6</u>	⁶⁷⁸ 6	⁶⁷⁸ 6
row6	²⁴⁸ <u>2</u>	²⁴⁸ 2	²⁴⁸ 2
row7	¹²⁴⁸ 24	¹²⁴⁸ 4	¹²⁴⁸ 4
row8	78	<u>78</u>	78
row9	5	5	5

-----BOX H-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row1	5	9	<u>8</u>	5	9	8
	1678		3678	1678		3678
row2		4	3	167	4	3
	1678			167		
row3		678	2	678	67	2
				167	67	

	COLUMN 4		
	start	step1	step2
row1	9	9	9
	24678	24678	24678
row2	<u>24</u>	24	24
	2468	2468	2468
row3	<u>24</u>	24	24
	467	467	467
row4		<u>67</u>	67
	678	678	678
row5			8
row6	3	3	3
row7	5	5	5
	1678	1678	1678
row8	167	<u>167</u>	167
	1678	1678	1678
row9	167	<u>167</u>	167

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 1 START	2	168 	3	9	678 <u>8</u>	678 67	1678 <u>1</u>	5	4
STEP 1	2	168 <u>6</u>	3	9	678 8	678 67	1678 1	5	4
STEP 2	2	168 6	3	9	678 8	678 7	1678 1	5	4

-----BOX B-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row1	9	678 8	678 <u>7</u>	9	678 8	678 7
row2	24678 24	1	4678 67	24678 24	1	4678 6
row3	2468 24	3	5	2468 24	3	5

After updates of row 6, col 7, box H, col 4, row 1 and box B:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	6	3	9	8	7	1	5	4
row2	46	5	9	24678 24	1	4678 6	678 78	3	278
row3	7	468	148	2468 24	3	5	9	68 6	128
row4	8	9	2	467 67	67	1	3	467 4	5
row5	45	3	45	678 8	2	9	678 6	1	78 7
row6	1	7	6	3	5	48 4	248 2	248 8	9
row7	3	18 1	7	5	9	38 8	1248 4	248 2	6
row8	56	2	158	1678 167	4	3678 3	78	9	178 1
row9	9	468	148	1678 167	678 67	2	5	78 7	3

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

-----BOX A-----

	start			step 1			step 2		
	col 1	col 2	col 3	col 1	col 2	col 3	col 1	col 2	col 3
row1	2	168 <u>6</u>	3	2	168 6	3	2	168 6	3
row2	46	5	9	46	<u>4</u>	5	46	4	5
row3	7	468	148	7	468 48	148	7	468 <u>8</u>	148 18

	step 3		
	col 1	col 2	col 3
row1	2	168 6	3
row2	46 4	5	9
row3	7	468 8	148 1

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 3 START	7	468 <u>8</u>	148 <u>1</u>	2468 24	3	5	9	68 6	128
STEP 1	7	468 8	148 1	2468 24	3	5	9	68 6	128 <u>2</u>
STEP 2	7	468 8	148 1	2468 4	3	5	9	68 6	128 2

-----BOX B-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row1	9	678 8	678 7	9	678 8	678 7
row2	24678 24	1	4678 6	24678 2	1	4678 6
row3	2468 <u>4</u>	3	5	2468 4	3	5

	COLUMN 9	
	start	step1
row1	4	4
	278	278
row2		8
	128	128
row3	<u>2</u>	2
row4	5	5
	78	78
row5	<u>7</u>	7
row6	9	9
row7	6	6
	178	178
row8	1	1
row9	3	3

		col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 2	START	46 4			24678 2		4678 6	678 78		278 <u>8</u>
	STEP 1	46 4			24678 2		4678 6	678 7		278 8

-----BOX I-----

	start			step 1		
	col 1	col 2	col 3	col 1	col 2	col 3
row1	1248 4	248 2	6	1248 4	248 2	6
row2	78	9	178 1	78 8	9	178 1
row3	5	78 7	3	5	78 7	3

After updates of box A, row 3, box B, col 9, row 2 and box I:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	6	3	9	8	7	1	5	4
row2	4	5	9	2	1	6	7	3	8
row3	7	8	1	4	3	5	9	6	2
row4	8	9	2	467 67	67	1	3	467 4	5
row5	45	3	45	678 8	2	9	678 6	1	78 7
row6	1	7	6	3	5	48 4	248 2	248 8	9
row7	3	1	7	5	9	38 8	1248 4	248 2	6
row8	56	2	158	1678 167	4	3678 3	78 8	9	178 1
row9	9	468	148	1678 167	678 67	2	5	78 7	3

solved squares in black

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 9 START	9	468	148	1678 167	678 67	2	5	78 <u>7</u>	3
STEP 1	9	468	148	1678 16	678 <u>6</u>	2	5	78 7	3
STEP 2	9	468 48	148	1678 <u>1</u>	678 6	2	5	78 7	3
STEP 3	9	468 48	148 48	1678 1	678 6	2	5	78 7	3

	start	step1
row1	168 6	168 6
row2	5	5
row3	468 <u>8</u>	468 8
row4	9	9
row5	3	3
row6	7	7
row7	18 1	18 1
row8	2	2
row9	468 48	468 4

-----BOX G-----

start

	col 1	col 2	col 3
row1	3	18 <u>1</u>	7
row2	56	2	158
row3	9	468 <u>4</u>	148 48

step 1

	col 1	col 2	col 3
row1	3	18 1	7
row2	56	2	158 58
row3	9	468 4	148 <u>8</u>

step 2

	col 1	col 2	col 3
row1	3	18 1	7
row2	56	2	158 <u>5</u>
row3	9	468 4	148 8

step 3

	col 1	col 2	col 3
row1	3	18 1	7
row2	56 6	2	158 5
row3	9	468 4	148 8

COLUMN 1	
	start
row1	2
row2	46
	<u>4</u>
row3	7
row4	8
row5	45
	5
row6	1
row7	3
row8	56
	6
row9	9

		col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 5	START	45		45	678			678		78
		<u>5</u>	3		8	2	9	6	1	7
STEP 1		45		45	678			678		78
		5	3	4	8	2	9	6	1	7

COLUMN 5		
	start	step1
row1	678 8	678 8
row2	1	1
row3	3	3
row4	67 7	67 7
row5	2	2
row6	5	5
row7	9	9
row8	4	4
row9	678 <u>6</u>	678 6

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 4 START	8	9	2	467 67	67 <u>7</u>	1	3	467 4	5

STEP 1	8	9	2	467 6	67 7	1	3	467 4	5
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	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
ROW 8 START	56 <u>6</u>	2	158 5	1678 167	4	3678 3	78 8	9	178 <u>1</u>

STEP 1	56 6	2	158 5	1678 7	4	3678 3	78 8	9	178 1
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After updates of row 9, col 2, box G, col 1, row 5, col 5, row 4 and row8:

	col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8	col 9
row1	2	6	3	9	8	7	1	5	4
row2	4	5	9	2	1	6	7	3	8
row3	7	8	1	4	3	5	9	6	2
row4	8	9	2	6	7	1	3	4	5
row5	5	3	4	8	2	9	6	1	7
row6	1	7	6	3	5	4	2	8	9
row7	3	1	7	5	9	8	4	2	6
row8	6	2	5	7	4	3	8	9	1
row9	9	4	8	1	6	2	5	7	3

A check of all rows, columns and boxes shows them all to be 9-perfect, so this is the solution, and this unorthodox method has succeeded.